Gradient-index Optics

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Abstract

The history of man-made devices involving the use of gradients of refractive index is briefly outlined and a typical component, the Wood lens, is described. Current methods for producing controlled gradients are reviewed, together with their limitations and applications. Some possibilities for using gradients to improve the design of spectacle and contact lenses are discussed.

Abrégé

Ce travail fait l'historique de différents appareils optiques utilisant un index de réfraction gradué et s'arrête sur un, soit, la lentille Wood. Les techniques modernes servant à la fabrication de ces "index gradués" sont décrites ainsi que leur applications et leurs limites. On traite de la possibilité d'utiliser des index gradués dans la fabrication de nouvelles lentilles ophtalmiques et de lentilles ce contacte.

Introduction

It has long been known that gradients of refractive index play a significant role in nature. In the atmosphere such gradients are responsible for mirages and similar effects. They also occur in the eyes of many species, notably in our own crystalline lenses.[1,2] Nearly a century ago, Exner[3] hypothesised that the functioning of many superposition and apposition compound eyes must depend upon the existence of suitable radial index gradients in the lens cylinder of each ommatidium, a proposal amply confirmed by later studies with interference microscopes[4].

The earliest theoretical study to suggest that inhomogeneous media potentially offer advantages in man-made devices was that of Maxwell[5], who found that any object point lying within a medium having a specified index gradient would be perfectly imaged at a single point within that medium. Only the last decade, however, has seen a real growth of interest in the field, well described in lengthy reviews by Marchand[6,7] and in a recent collection of papers in the journal Applied Optics[8]. The theoretical design possibilities are now being vigorously investigated and considerable progress has been made in devising suitable techniques for the production of the desired gradients. This review outlines the current position of this young technology and offers some speculations to indicate its potential in the optometric context.

The Wood Lens

As an introduction to the ideas of gradient-index optics, it will be helpful to compare the characteristics of a lens of uniform refractive index, n, with its possible gradient-index counterpart, the Wood lens[9].

One way of describing the effect of the thin, homogeneous, plano-convex lens in air, illustrated in fig 1a, is in terms of its action on an incident wavefront. Recalling that the time of passage of each local element of the wavefront through the lens is proportional to the corresponding optical path (ie. the product of the geometrical thickness and the refractive index), if the lens is to be convergent the optical path must decrease with distance from the axis in such a way that a plane, incident wavefront is converted to a spherical, convergent, transmitted wavefront.

Using the symbols of fig 1a and the small-angle, paraxial approximation, which implies that there is negligible change in incidence height on passing through the lens, and that $x \ll y \ll f^1$, the sag formula for the transmitted wavefront which has just emerged from the posterior pole of the lens gives

$$x = \frac{y^2}{2f^1}$$

where f^1, the radius of curvature of the transmitted wavefront, is the second focal length of the lens. As the

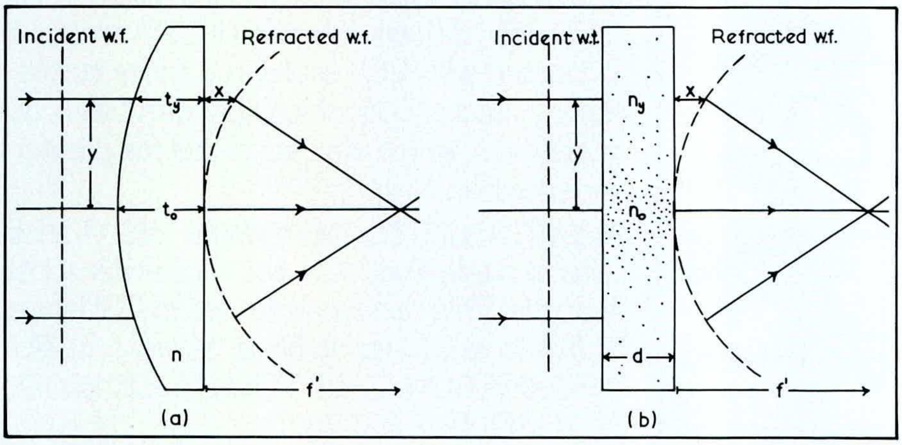

Fig 1 (a) The effect of a homogeneous plano-convex lens on a plane incident wavefront.

(b) The effect of a Wood lens, having a radial or cylindrical gradient of refractive index, on a plane incident wavefront. Stippling represents the region of higher index. In both cases the relative thickness of the lens has been greatly exaggerated for clarity.

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optical paths between corresponding portions of the incident wavefront tangential to the anterior pole of the lens and the emergent wavefront must be equal

$$nt_o = nt_y + (1.0).[(t_o - t_y) + x]$$

where the refractive index of the air is 1.0.

$$\text{or } t_o - t_y = \frac{x}{n-1} = \frac{y^2}{2f^1(n-1)} \quad \ldots\ldots \text{(i)}$$

Equation (i) shows that the thickness of the lens must decrease in parabolic fashion with distance from the axis. It is, of course, a form of the sag formula for the anterior surface of the lens, implying, not surprisingly, that this surface must be spherical with radius, r, given by

$$r = f^1(n-1).$$

Consider now the lens shown in fig 1b. This has constant thickness, d, but a radial, or cylindrical, gradient of refractive index such that the index, n_y, changes systematically with distance, y, from the lens axis; the iso-index surfaces are thus cylindrical. Such a lens is now usually called a Wood lens, after its originator[9]. Again, if the emergent wavefront is to be spherical with radius f^1, we have

$$x = \frac{y^2}{2f^1}$$

Further, the requirement that the optical paths between corresponding points on the incident and emergent wavefronts be equal implies, if n_o is the index on the lens axis,

$$n_od = n_yd + (1.0).x$$

$$\text{whence} \quad n_o - n_y = \frac{y^2}{2df^1} \quad \ldots\ldots\text{(ii)}$$

Comparison of equations (i) and (ii) shows that the parabolic thickness variation of the homogeneous lens has been replaced by a parabolic index variation in the constant-thickness, gradient-index case. This is again a paraxial approximation, more refined derivations being given by Wood[9] and Fletcher et al[10].

Wood, using mixtures of glycerine and gelatine, was able to successfully demonstrate, at the beginning of this century, the imaging properties of lenses of this type. An index which decreases with distance from the axis gives a converging lens, while one that increases with distance gives a diverging lens.

Gradients Used and Related Devices

From the foregoing, it is obvious that the use of an index gradient allows the optical path to be varied independently of the geometrical thickness. It thus offers a whole new degree of freedom to the optical designer. In principle, any three-dimensional variation in index can be conceived but practical constraints have so far led designers to concentrate on three basic types of gradient - axial, radial or cylindrical, and spherical (fig 2).

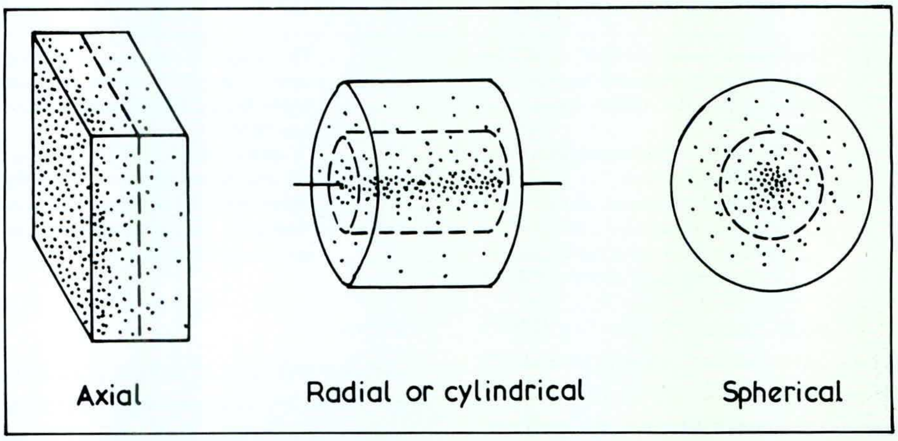

Fig 2 Gradients used in most types of device that have been considered up to the present time. Stippling qualitatively represents the regions of higher index and the dashed lines show typical iso-index surfaces. Gradients of opposite sign are, of course, also possible.

In the axial case, the iso-index surfaces are planes perpendicular to the optical axis. Sands[11] and Moore[12] have shown that there is a one-to-one correspondence between the use of an axial gradient and the use of an aspheric surface of revolution. Thus, a lens whose design in homogeneous index material requires an aspheric surface can be replaced by one with spherical surfaces and an appropriate axial gradient, with obvious simplification from the surfacing point of view. Moore[13,14] gives examples of the design of a singlet with two spherical surfaces, an axial gradient being used to control spherical aberration and coma.

Radial, or cylindrical, gradients have already been discussed in the context of the Wood lens. Marchand[7,15] has investigated the potential of Wood lenses with plane surfaces as photographic singlets and has calculated the corresponding aberrations. He finds that, providing the specified index profile can be made with sufficient accuracy, good control of aberration is possible. Further refinement can be achieved if spherical or aspherical surfaces are used in conjunction with the index gradient.[15] Other potential designs have been considered by Moore.[13,14] Some prototype lenses of this general type, possessing reasonable apertures, have been made at the University of Rochester.[16]

The biggest usage so far of radial gradients has been in the form of fibres for optical communication purposes, having diameters $\lesssim 100$ μm.[17] In such fibres, light is propogated in a series of S-shaped paths, never touching the fibre boundaries and propogation losses can be very low. Increasing use is also being made of GRIN (gradient index) rods.[18] These are typically several mm in diameter and normally have a parabolic index gradient which produces a series of inverted and erect, unit-magnification images down the length of the rod (fig 3). The length or 'pitch' of the rod, L, required to produce an image with unit positive magnification will vary with the parameters of the gradient. By cutting the rods to lengths L/4, L/2, L etc collimators, invertors or image relays can be produced. The action of these GRIN rods is, in fact, exactly the same as that proposed by Exner[3] for the lens cylinders of compound eyes.

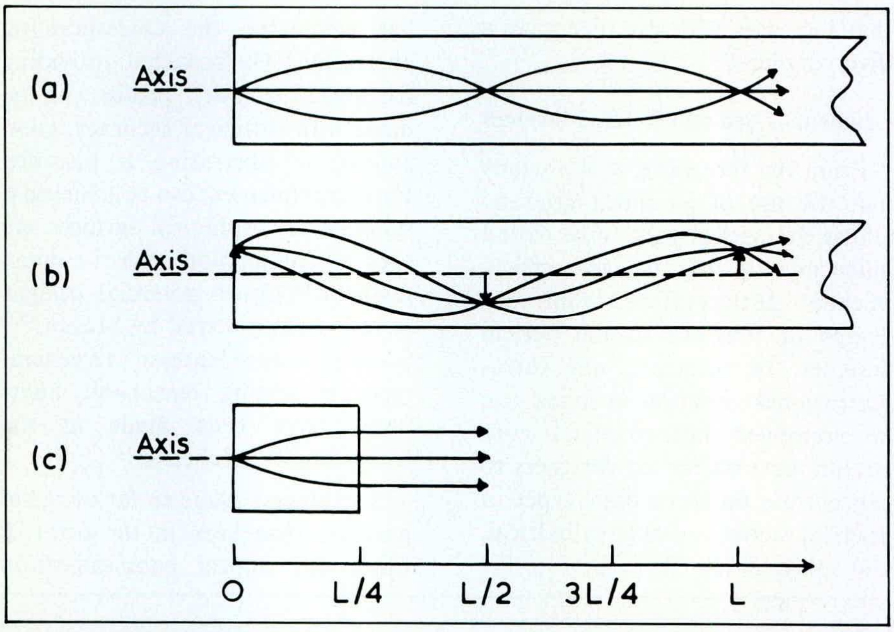

Fig 3 Meridional cross-sections of GRIN (gradient index) rods, having approximately parabolic radial index gradients.
(a) Ray paths from an on-axis point on the entry face.
(b) Ray paths from an off-axis entry point showing how a sequence of inverted and erect, unit-magnification images is produced down the length of the rod, repeating over a length L which depends upon the parameters of the index gradient.
(c) A quarter-pitch (L/4) GRIN rod, or lens, which collimates the light from any point on its entry face. A half-pitch (L/2) rod would act as an image invertor.

A wide range of uses for GRIN rods are under development[19,20], particularly as components in optical-fibre communication systems. One commercial photocopier already incorporates a GRIN-rod array to produce an erect unit-magnification image onto its copying drum[20,21]; again it is interesting to note that the optics of this system are closely analogous to those of the superposition compound eye. A further intriguing application, which may be of value in future ocular studies, is in the rigid 'needle scope'[18], designed particularly to allow examination of small internal regions of the body (fig 4). The GRIN rod, 0.75 mm in diameter, in the prototype instrument, is contained in a hollow needle. While the device lacks the flexibility of conventional fibrescopes, where the image is shared between a bundle of fibres, its small overall diameter gives it unique advantages, although problems remain to be solved as far as the image quality is concerned. There seems little doubt that GRIN-rods will be used in an increasing variety of instruments, including those intended for ophthalmic applications.

In the last type of simple gradient, the iso-index surfaces are spherical shells. While this was the earliest gradient to be considered, by Maxwell[3] and later by Luneberg[22], it has received little attention for optical purposes in recent years, although applications do exist in the field of microwave optics.

One rather separate application of index gradients involves their use as an anti-reflection treatment for optical surfaces, the overall extent of the gradients involved typically being only a few wavelengths. As the direction of these gradients is perpendicular to the surface being treated, they may be classified as axial or spherical, depending upon the surface geometry. As is well known, reflection at an interface between two optical media occurs because of the difference between the indices of refraction on either side of the boundary. This may, of course, be controlled by quarter-wavelength interference coatings but these require careful monitoring of film thickness and are not suitable for all substrates. Gradient index techniques reduce unwanted reflections by effectively replacing the abrupt discontinuity in refractive index at the boundary with a smooth gradation. Typically, then, the ideal gradient might involve a gradual increase in index from the air value of 1.0 to that of the bulk optical material (eg. 1.5). The extent to which the surface reflection is suppressed varies somewhat with the characteristics of the gradient[23,24] but reflectance at an air-glass interface can be reduced from $\sim$ 4% to less than 0.1%.

Techniques for Manufacturing Gradients

The range of devices that can be achieved in practice depends crucially upon the manufacturer's ability to reproduce the gradients specified by the designer. In imaging devices very careful control of the gradients is necessary if image quality is not to be impaired; in gradient index fibres for communications purposes a wider tolerance on gradient quality can be allowed.

Table 1 (after Moore[16]) lists the

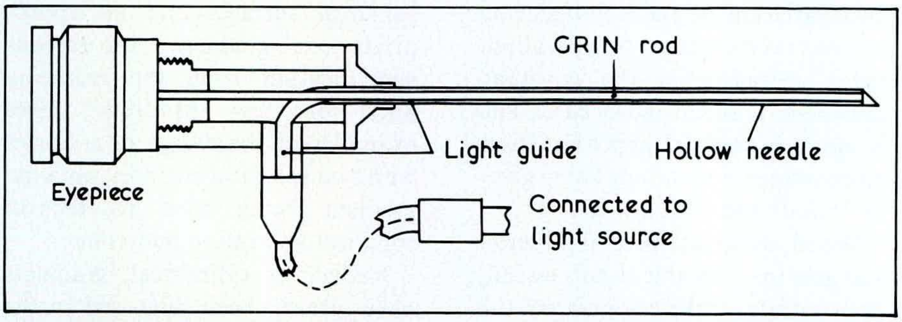

Fig 4 Schematic diagram of a 'needle scope' (after Uchida et al[18]), using a long GRIN rod contained in a hollow needle to relay an image to the eyepiece. The main problem with such devices at the present time is chromatic aberration and the difficulty in producing a uniform parabolic index gradient over the full length of the rod.

Method	Extent of gradient (mm)	Total index change
Neutron irradiation	0.1	0.02
Chemical vapour deposition	0.1	0.01
Ion exchange	10	0.04
Ion stuffing	50	0.04
Polymerisation techniques, etc.	100	0.03
Crystal growing	20	0.05
Anti-reflection treatments	0.001	0.50

Table 1: Methods for producing gradients of refractive index with approximate characteristics of resultant gradients (based on Moore[16]).

main techniques that have so far been devised. Their performance is specified in terms of two parameters: the overall depth of the zone of changing index and the overall change in the value of the index. It is immediately obvious that no method can yet produce large changes in index over reasonable distances.

In neutron irradiation, a boron-rich glass is bombarded with roughly-collimated thermal neutrons, the local neutron dose being controlled by a cadmium mask or attenuator of suitably varying thickness. In principle, the merit of this technique is that changes in the mask geometry allow a wide variety of gradients, typically a combination of axial and radial, to be achieved. Sinai[25] has demonstrated experimentally that the spherical aberration of a singlet can be corrected by the use of a neutron-induced index gradient; the index changes were found to be stable over periods of at least a few years. Nevertheless, only small index changes can be generated and it is difficult to see the method being used in a mass production context.

The chemical vapour deposition method, available in several variations[26], is widely used for the production of fibres with radial gradients. Typically, the starting material is a heated quartz tube, which is rotated about its axis. Through this tube are passed silicon chloride gas and oxygen, with controlled amounts of boron and germanium chloride vapours. Reaction occurs and concentric layers of doped glass build up on the inside of the tube, their refractive index being gradually changed by variation in the proportions of the reactants; the index of the deposited glass increases with the $Ge0_2$ content. When the concentric deposits have accumulated to provide the desired, usually parabolic, index profile, the tube is further heated and collapses to a rod. This 'preform' can then be drawn out in a suitable furnace to produce the gradient-index fibre. Strictly, the production process tends to give a step-index profile, with, perhaps, 1000 layers. These steps become an effective continuum when the preform is drawn to fibre diameters but make the method inapplicable to the production of larger components.

The ion-exchange method, which closely resembles the chemical process for toughening glass, is probably the most popular of the current methods for producing larger components with index gradients. It relies on diffusion, the base glass being immersed in a high-temperature (~600°C) bath of molten salt for some hours. During this period, cations from the bath exchange one-to-one with alkali ions from the glass. The salt might be, for example, lithium bromide with sodium atoms in the glass being replaced. For so-called Selfoc lenses having a radial gradient, K+ ions from a potassium nitrate bath replace T1+ and Na+ ions from the original glass rod.[27] The gradient produced is usually approximately parabolic, although it is to some extent controllable through changes in such parameters as the materials used and the temperature and duration of the process. Too high a temperature may lead to a deformation of the glass. It may be possible to speed the production process by the application of external electric fields, this being known as field-assisted ion exchange.[28]

Ion stuffing is, as table 1 indicates, a promising technique in terms of the extent and index range of the gradients that it can produce but, as yet, it remains in its infancy. The process involves the use of a glass which phase-separates on heating. In one variation[20] an alkali-borosilicate rod is used, which separates into alkali-rich and silica-rich phases. The alkali-rich phase is dissolved out, using weak hydrochloric acid, to leave a pure porous silica skeleton. This is now soaked in an aqueous dopant solution containing suitable ions at an elevated temperature to raise the refractive index. After uniform 'stuffing' has been achieved, the concentration of dopant is modified by placing the rod in a weaker solution of dopant, when ions now diffuse out of the outer regions of the rod ('unstuffing') to give a lower index. When the desired index profile has been arrived at, the remaining dopant solution within the silica skeleton is crystallised by a sudden drop in the temperature. Subsequent reheating and consolidation thus yield a rod with a radial index profile. Modification of the geometry of the original host glass could presumably generate other forms of gradient. Advantages claimed for this technique include rapid profiling, the ability to use high concentrations of a variety of dopants to give different index changes, the possibility of producing large components and the durability and reproducibility of the gradients achieved.

Several methods for producing plastic gradient-index materials have been discussed in the literature.[7,16,20,29,30] Such methods include partial polymerisation of an organic monomer by controlled irradiation with UV or laser light, which potentially can produce large components with a wide range of index gradients,[16] and exchange-diffusion between different monomers,[20,29,30] this being reminiscent of the techniques employed with glass. While plastic materials may well be of great interest in the future, as yet they remain at the relatively early experimental stage.

Crystal-growing methods seem at present to be likely to be most useful for infra-red components where normal glasses lose their transparency. Typically, crystal-growing might involve starting with a seed crystal of sodium chloride and an aqueous solution of sodium and silver chlorides. As the crystal is slowly pulled from the bath, it initially grows by adding further sodium chloride, thereby decreasing the concentration of the latter in the bath and increasing the relative concentration of silver chloride. Further growth of the crystal necessitates it taking some silver chloride into its structure, this proportion gradually increasing with time to produce a gradual change in crystal index. Moore[16] believes that it will soon be possible to produce combinations of gradient-index silicon and germanium for infra-red components.

Techniques for producing gradients for the reduction of surface reflections form a specialised subgroup. Early methods, reviewed by Macleod[31], involved decreasing the surface index of glass by etching. Since that time, three variations have emerged. In the first[32], a chemical etch-leach process is applied to glasses sensitised by a phase-separating heat treatment. This produces a porous film which has very low density on the air side, the density steadily increasing towards the substrate. Such a film produces very little scattering providing that the pores are sufficiently small compared with the wavelength of light. A second alternative involves covering the surface with a regular array of conical nipples, spaced ~200 nm apart. Naively, this produces a situation in which the fractional area presented to an incoming light beam by the higher index medium increases steadily from 0 to 100%: hence if the structure is small compared with the incident wavelength, the incoming beam sees a steady increase in effective refractive index. Remarkably, this means of reducing reflection was first hypothesised by Bernhard[33] as occurring on the corneas of nocturnal moths and the efficacy of a similar man-made nipple array, produced by a crossed system of interference fringes in photoresist, was demonstrated by Clapham and Hutley.[34] The last method, which bears some similarities to the nipple array, is applicable to plastic surfaces. It involves bombardment of the surface with high energy ions and subsequent etching of the particle tracks[24]. With a proper choice of parameters, the roughly conical etch pits overlap to produce a surface topography such that, on the wavelength of light, the material appears to have smoothly changing mass density and refractive index through the surface layer of a few wavelengths thickness. Both the ion-etch method and the nipple array have been demonstrated to achieve reflectances $< 0.1\%$, comparable with the best commercially-available, multilayer, interference coatings. While these techniques might have application to, for example, large or flexible components, particularly in solar energy technology, they are unlikely to be as robust as interference coatings and the latter therefore seem unlikely to be supplanted in the context of ophthalmic lenses.

To summarise, while chemical vapour deposition and ion-exchange techniques are now well-established methods for producing specific types of relatively small components, much remains to be done before any desired index gradient can be fabricated in any chosen size of component. Problems also remain in measuring the characteristics of the gradients achieved, although interferometric[35], prism[36], moiré[37] and other methods have been proposed; the chromatic variation of the index[38] is a particularly important parameter in many imaging applicators. However, great progress has been made in the last decade and it seems reasonable to expect that similar advances will be made in the future.

Ray-Tracing With Gradients

Clearly, the well-tried methods for tracing rays through homogeneous media, where the rays travel straight line paths from surface to surface, are inapplicable to gradient-index media, where such paths are in general curved. It becomes necessary to follow the ray along a series of short elemental paths and to describe its changes in direction by means of differential equations which can be derived from Fermat's principle.[7] Details of ray-tracing procedures are beyond the scope of this review but encouraging progress in developing methods suitable for computer programming has been described.[6,7,20]

Optometric Applications

It has been seen that, in principle, availability of index gradients presents the lens designer with a whole new degree of freedom. This might be particularly significant in optometry, where practical constraints normally limit the designer of single-vision lenses to permutations of only two surfaces and a single index and lens thickness, unlike the more fortunate camera-lens designer who can indulge in multi-element devices where many surfaces, lens separations, thicknesses and indeces allow better aberration control.

On the other hand, it is evident that limitations of the effectiveness and cost of present methods of producing gradients set constraints on the feasibility of many potential optometric applications that might come to mind. With the rapid advance of production techniques it is, however, well worthwhile to consider possible situations in which the lens gradients might be attractive.

Single-Vision Spectacle Lenses

As already noted, an axial gradient can be used to replace an aspheric surface of revolution.[11,12] Surfaces of the latter type have been used by

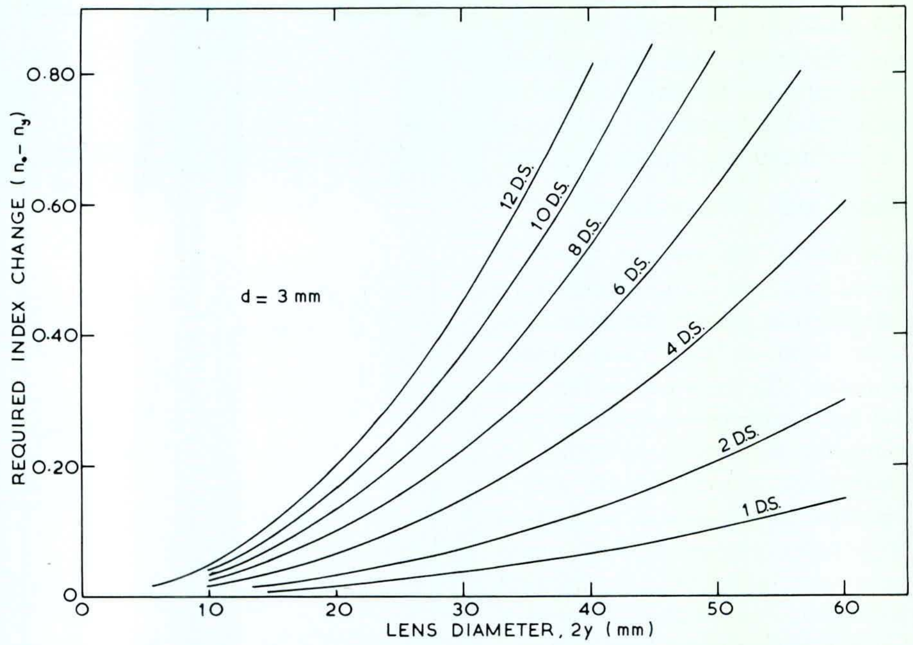

Fig 5 Required change, $n_o - n_y$, in refractive index between axis and edge as a function of the diameter of a Wood lens for the spherical lens powers indicated. It is assumed that all the lenses have a uniform thickness of 3 mm.

several ophthalmic lens designers in recent years to produce lenses which are thinner or have superior cosmetic or optical properties. In general, however, aspheric surfaces cost more to produce than spherical surfaces. It may be, then, that useful improvements in overall lens characteristics could be achieved by adding an axial gradient to ophthalmic lenses with spherical surfaces. It might be possible to reduce costs by applying the gradient simultaneously to the front surfaces of large batches of lens blanks, perhaps by the ion-exchange process. If such gradients extended over a relatively short axial distance, they would be unaffected by any subsequent surfacing operations at the rear of the lenses.

Whitney[39] has outlined in a recent patent how a radial gradient might improve the optical performance of an ophthalmic lens of meniscus form; substantial improvements in performance over homogeneous lenses are claimed.

A further area in which it is tempting to consider the use of index gradients, in this case of the radial or cylindrical type, is for full-aperture high-powered ophthalmic lenses. When made in homogeneous material, such lenses inevitably possess high centre or edge thicknesses, with consequent disadvantages in weight and appearance; high-index glasses can reduce but not eliminate this problem. Could a Wood lens in either plano or curved form be superior to its homogeneous counterparts?

It will be recalled from equation (ii) that the power of the Wood lens is given by

$$F = \frac{1}{f'} = \frac{2d}{y^2}(n_o - n_y) \qquad .....(iii)$$

The constraints to the lenses that can be produced in practice are evidently set by the maximum extent and index change of the manufactured gradients (table 1). If we require some particular power F, thickness d, and diameter 2y, this implies that a specific value of ($n_o - n_y$) needs to be achieved. It can further be seen that the necessary value of ($n_o - n_y$) increases with the square of the lens diameter, unfortunately implying particular problems in the production of full-aperture lenses. Fig 5 illustrates this difficulty for the case where d has been assigned the relatively large value of 3mm. It is clear that production of lenses of reasonable power ($\gtrsim$6DS.) and aperture ($\gtrsim$50mm) requires substantial index changes ($\gtrsim$0.6). Table 1 emphasises that the latter are well beyond the capabilities of present processes which can only achieve maximum index differences of $\sim$0.04. Thus there is no immediate prospect of high-power, full-aperture, single-vision, Wood-type lenses becoming viable for optometric use.

It is worth noting that further problems arise if thickness variation is introduced into a Wood lens. Thus, while it might naively be thought possible to grind a cylindrical correction onto one surface of the lens, the resultant thickness variation automatically introduces a variation in the power conferred by the radial gradient. The local power in any meridian becomes equal to the sum of the surface power, conferred by the curvature of the surfaces and the local index, and the Wood lens power, dependent on the index gradient and the local thickness.[40] If then, the latter varies, there is an associated variation in the correcting power of the lens over its surface.

In spite of these difficulties, Wood-type lenses of reduced aperture are very nearly feasible at the present time. An index difference $n_o - n_y$ = 0.04 would allow an 8mm diameter lens of power 15D and thickness 3mm to be produced, while the diameter could increase to nearly 13mm if an index difference of 0.10 could be achieved. Thus 'invisible' gradient index lenticulars could be envisaged.

A small diameter, parabolic-index rod would be inserted into a concentric outer hollow cylinder of uniform material. The lenses could then be 'sliced off' from this rod, variation in thickness giving variation in spherical power. The lens could, of course, be in curved rather than in plano form, providing its thickness remains constant. The problem of incorporating a cylindrical prescription could be overcome by making the lens in 'two-layer' form, the gradient index element providing the anterior component with spherical power, while the rear element of constant index has a toroidal rear surface to correct ocular astigmatism and, possibly, to supplement the spherical power (fig

6a). Possible advantages in weight and thickness over conventional lenticulars are critically dependent upon the development of techniques for producing larger index gradients.

Bifocal and Varifocal Lenses

A bifocal lens with an 'invisible' Wood addition could evidently be fabricated in exactly the same two-layer form as the gradient-index lenticular (fig 6b), where in this case the homogeneous rear element provides the distance prescription. A 1mm thick Wood addition with a diameter of 10mm and an axis-to-edge index change of 0.04 would have power of 3.2 D.S., so that such lenses are almost within the range of current technology. Any potential advantage of such an invisible bifocal would have to be balanced against the cost and difficulty of producing it in comparison with existing types.

More interesting is the possibility of producing varifocal lenses, having a smooth gradation of power between the distance and reading portions, by gradient-index techniques. Recalling that the power of a lens can be attributed to the variations in optical thickness across its area, it is possible to envisage a lens of constant thickness where variations in local refractive index are used to produce any desired change in optical thickness. Thus the complex aspherics of such lenses as those of the Varilux or Unison type would be replaced by spherically-surfaced lenses with complex index variations.

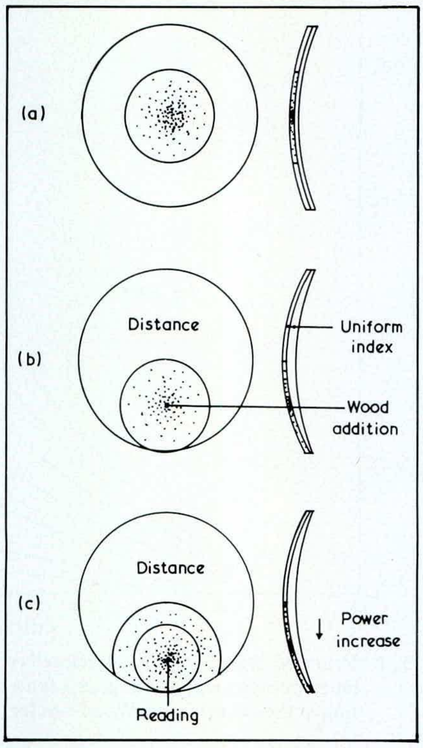

Fig 6 Some hypothetical ophthalmic lens designs incorporating elements with radial refractive index gradients. In each case, the problem of correcting ocular astigmatism makes it necessary for the lens to be made in a two layer form, astigmatism being corrected by grinding an appropriate toroidal surface on the rear, homogeneous component. This would not be necessary for purely spherical corrections. Stippling represents regions of higher index.

(a) An 'invisible' gradient-index lenticular. A near-afocal carrier of homogeneous index material supports the radial-gradient, Wood spherical correction.

(b) An 'invisible' gradient-index bifocal. The distance prescription is in homogeneous material and the Wood lens provides the addition.

(c) A concentric varifocal lens. The reading portion has a parabolic index gradient while the radial gradient in the intermediate zone is of a form to give a gradual blend of sagittal power between that of the reading and distance portions of the lens.[40,41]

It should be noted that, in principle, any concentric variation in sagittal power can be achieved through the use of an appropriate radial gradient[41], although the feasibility of producing such gradients remains unproven.

Whether the required three-dimensional changes in index might be created in practice is debatable, but a concentric varifocal of the type illustrated in fig 6c could be constructed in the foreseeable future. The outer annular distance portion would be of uniform index and the central, circular region, constituting the reading portion to provide the appropriate addition, would be of the familiar Wood lens form, with a radial, parabolic index gradient. In the intermediate annular zone, the aim would be to achieve a smooth progression of power between the distance and reading portions of the lens. The required index gradient, which depends on terms of higher order than y^2, can be calculated in straightforward fashion.[41] Unfortunately, like the much earlier Beach 'blended bifocal'[42], where the required concentric variation in optical thickness was achieved by varying the geometrical thickness of a lens of uniform index, such a concentric varifocal suffers from unavoidable unwanted cylindrical power in the transition zone[43], which may limit its usefulness. It can be shown[41] that this unwanted cylindrical power is given by $y.dF_S/dy$, where y is the zonal radius and dF_S/dy is the radial gradient of sagittal power in the transition zone. Thus the problem becomes worse if the reading zone is made large, since this inevitably increases y in the transition zone.

Another possible design of varifocal using an index gradient and a carrier lens has been outlined in a patent.[44] Large changes in index (~ 0.14) were envisaged and no predictions were made of any associated aberrations.

Contact Lenses

There might well be fitting advantages in using a contact lens that had constant thickness across its area. Is this, then, a possible role for a gradient-index lens of the Wood type? Again we must consider the possible parameters. Taking a lens thickness $d = 0.1$ mm, a lens diameter $2y = 8$ mm and a nominal power $F = +5D$, we find that the required axis-to-edge index change for a Wood lens (equation ii) is 0.40. This is well beyond the scope of present technology (table 1). Moreover, if the lenses were cut from a single rod of constant radial gradient, each lens power would demand a different thickness of material, which seems undesirable. The alternative, of having a different parabolic, radial-gradient, stock rod for each power seems extravagant in terms of manufacturing demands.

At present, then, contact lenses in Wood form seem unlikely to be viable. It is possible that, if comparatively crude optical performance is acceptable, radial gradients could have a role in so-called variable focus bifocals, where the contact lens power changes progressively from the centre towards the periphery.

Discussion

It seems clear that work currently in progress is likely to ensure a permanent place for components with controlled index gradients in optical instrumentation. Recalling that, more than two decades after the original laboratory demonstration of the potential of fibre optic components, these are now beginning to find their way into optometric instrumentation, it seems likely that gradient index components may follow a similar history. More problematical is the extent to which index gradients may be helpful in spectacle and contact lenses. It is important, then, that the theoretical improvements that might be achieved be properly explored at this early stage, to provide incentives for the practical production of the required gradients. This promises to be an interesting area of work during the next decade.

Addendum

Since this article was prepared, a further valuable collection of papers on the topic of gradient-index optics has appeared in the journal 'Applied Optics'[45]. Among the many interesting developments described in this collection, particularly striking is the production of very large index changes (~ 0.22) in a TiF_6 Schott glass, albeit only over the short distance of 0.04 mm[46]. As already noted, large index changes of this order would be needed for the successful production of Wood-type ophthalmic lenses.

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