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Price Regulation and Health Care with Disease Dynamics

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We build up a differential game to investigate the interplay between the quality of health care and the presence of an evolving disease in a duopoly where patients are heterogeneous along the income dimension. We study the Markov perfect equilibrium, and we identify the admissible parameter region wherein price regulation achieves the twofold objectives of ensuring cares to all patients and heal all of them.

Keywords: health care markets, quality, dynamic games

JEL Classifications: C73, H42, I11, I18, L13

1 Introduction

So far, the theoretical literature on the quality of health care has extensively dwelled on the analysis of the provision of health care in the framework of multidimensional product differentiation combining vertical and horizontal dimensions (see Barros and Martinez-Giralt, 2002, Beitia, 2003, Brekke *et al.*, 2006, Brekke *et al.*, 2010, Bardey *et al.*, 2012, *inter alia*). However, a feature common to all the contributions belonging to this stream of research is that patients are supposed to be identical in their capacity of paying for medical cares, i.e., this approach generally leaves aside the typical assumption of differences in willingness to pay (income) inherent to the analysis of quality choice in the theory of industrial organisation, dating back to Spence

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(1975) and Mussa and Rosen (1978).[1] Additionally, the provision of health care is investigated in settings where the objective of health care, i.e., the disease, is only implicitly considered, so that these models could be interpreted as describing, in general, differentiated industries where firms are typically subject to price regulation.

Here, we set out with a twofold aim: to study the design of a health care regulation system where (i) the existence of a disease is explicitly accounted as a dynamic process, and (ii) patients differ in income and therefore also in their resulting willingness to pay for medical treatment.

We build up a dynamic duopoly with vertical differentiation where a high and a low-quality hospital set their health care levels, prices are regulated by a public agency, and the market can be partially covered, in the sense that some individual may not receive health care. The assumption of an health care system where prices are regulated but different qualities are available can be interpreted in several ways. On the one hand, this may be justified by the imperfect information patients have concerning the quality of the competing hospitals (Gaynor, 2006). Another interpretation is a geographical argument, namely, the trade-off between the quality of health services and travel costs, even if the geographical access to health care is not explicitly modelled (Pope, 1989). Finally, this assumption may represent a mixed health system where the price of privately provided health is regulated (examples are the health systems in the United Kingdom and Italy).

In the model, hospitals set their quality level non cooperatively at every instant. The demands for high and low quality differ depending on patients' income. Finally, we interpret price regulation in this model as a fine tuning device whereby the public agency adjusts prices so as to attain two goals: (a) ensure universal treatment, i.e., full market coverage, and (b) completely heal all patients.

Our approach is suited to represent some relevant payment methods for health care, such as "fee-for-service". This is a method in which doctors and other health care providers are paid for each service performed. Examples of services include individual visits or clinical activities (injections, lab tests, x-rays, and so forth). It is commonly adopted, with regulated prices, in many OECD countries, such as the U.S. Medicare and Medicaid programmes, or most European health care systems.

[1]Mougeot and Nagelen (2013) consider vertical differentiation and quality competition in health care markets. While they focuss on a static setting with symmetric hospitals with free entry, in the present paper we investigate on a dynamic setting with hospital differing in levels of health care.

Our results show that there is a unique steady state equilibrium where hospitals provide vertically differentiated services. Then, we show the conditions according to which there exists a unique vector of regulated prices ensuring indeed that every individual is cared for and the disease has completely disappeared. Our analysis yields a clearcut message as to the design of the price regulation policy, as there exists a parameter range wherein price regulation simultaneously delivers two eggs in one basket, namely, universal service and the elimination of disease.

This paper is mainly related to the theoretical literature on quality competition in health care and regulated markets in dynamic settings (Brekke et al., 2010, Brekke et al., 2012 and Siciliani et al., 2013, inter alia). In particular, Brekke et al. (2010) offer the first analysis of quality competition in health care markets with regulated prices with a differential game approach. We contribute to the literature by explicitly analyse a disease as dynamic process, and by assuming patients being heterogeneous in willingness to pay for medical treatment.

Our analysis is also related to the literature of disease eradication (see Anderson and May, 1991, Geoffard and Philipson, 1997, Barrett, 2003 and Barrett and Hoel, 2007, *inter alia*). However, while this literature examines the effects of vaccination, this paper focusses on hospital treatment.

The remainder of the paper is structured as follows. The setup is laid out in section 2. Section 3 describes the Markov perfect equilibrium, whereas Section 4 shows the price regulation policy. Concluding remarks are in section 5.

2 The model

We adopt a continuous time setup, where time is $t \in [0, \infty)$. To investigate the optimal provision of health services, we rely on a variant of the vertical differentiation model with hedonic preferences originally due to Mussa and Rosen (1978), where individuals are indexed by a marginal willingness to pay for health care $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, $\underline{\theta} > 0$.[2] Over such interval, the population is uniformly distributed with unit density, so that its size is equal to one and the proportion of each type with respect to the overall population remains unchanged over time. Two hospitals serve the market, each one being characterised by a different quality level. Health care may be intepreted as physician office visits or clinical activities, such as injections.

[2]The marginal willingness to pay can be interpreted as the reciprocal of the marginal utility of income, whereby, if the latter is decreasing, θ increases in income. See, e.g., Tirole (1988, ch. 2).

The instantaneous measure of health care quality is $q_i(t)$, $i \in \{L, H\}$, which is the control variable of hospital i, where $q_H(t) \geq q_L(t) \geq 0$. The price of quality i at any time t is constant at p_i, being regulated by the government or a public agency. For reasons that will become clear in the remainder, we allow for $p_H \geq p_L \geq 0$.

By accessing the high-quality health service, an individual of type θ attains the following net instantaneous surplus:[3]

$$U_H(t) = \theta + q_H(t) - p_H - D(t), \tag{1}$$

where $D(t) \geq 0$ measures the level or intensity of disease suffered by this individual at time t. Instead, if the same individual resorts to the inferior quality care, the resulting net surplus is:

$$U_L(t) = k\theta + q_L(t) - p_L - D(t),$$

where $k \in (0, 1)$ is a positive time-invariant parameter capturing the idea that gross satisfaction from receiving the low quality is lower. The third and last admissible case is that where an individual does not receive any health care; for the sake of simplicity, we set the corresponding utility level to zero - which does not necessarily correspond to the death of the patient. To make sense of this normalisation, consider what follows. Parameter θ measures the willingess to pay for medical care. If the patient is not being served, one may imagine that he/she receives some form of parental care at home, resulting in an alternative utility $n(t) - D(t)$, where $n(t)$ measures the instantaneous amount of the (unmodelled) parental care. It suffices to assume that $n(t) = D(t)$ at all times to economise on the number of endogenous variables.

In line of principle, we admit the possibility that the poorest section of the population be priced out of health cares. Accordingly, we set up the model under partial coverage. Although it would perhaps be natural to assume full coverage (as, e.g., in Brekke *et al.*, 2010), we refrain from imposing it at the outset as it would cause the disease $D(t)$ to disappear from the profit function hospital L. This, in turn, would trivially imply that firms should be taxed in order to induce them to internalise this crucial aspect of their service.

In order to construct the demand functions for the high- and low-quality services, we solve

[3]This specification of the utility function is borrowed from Colombo and Lambertini (2003).

the indifference conditions:

$$U_H(t) = U_L(t) \Leftrightarrow \widehat{\theta}(t) + q_H(t) - p_H - D(t) = k\widehat{\theta}(t) + q_L(t) - p_L - D(t) \tag{2}$$

yielding

$$\widehat{\theta}(t) = \frac{p_H - q_H(t) - p_L + q_L(t)}{1-k} \tag{3}$$

and[4]

$$U_L(t) = k\widetilde{\theta}(t) + q_L(t) - p_L - D(t) = 0 \tag{4}$$

yielding

$$\widetilde{\theta}(t) = \frac{D(t) + p_L - q_L(t)}{k}. \tag{5}$$

Hence, the demand functions are:

$$x_H(t) = \overline{\theta} - \widehat{\theta}(t)\,;\; x_L(t) = \widehat{\theta}(t) - \widetilde{\theta}(t)\,. \tag{6}$$

On the supply side, we describe a health sector consisting in private hospitals behaving as profit-seeking units. Prices are being regulated over the entire horizon of the game by a public agency, whose objectives will be discussed in detail in the remainder. Hence, hospital i controls only the quality of its services $q_i(t)$ over time. The supply of health care entails the instantaneous cost $\Gamma_i(t) = cx_i^2(t)$, while any other costs are assumed away. We disregard the possible presence of a lump-sum transfer to hospitals, as it is altogether immaterial in terms of the characterisation of equilibria and the viability of firms. Therefore, the hospital's instantaneous profit function is:

$$\pi_i(t) = [p_i - cx_i(t)]\,x_i(t)\,. \tag{7}$$

The disease dynamics is represented by the following state equation:

$$\dot{D} = sD(t) - v[q_H(t) + q_L(t)]\,, \tag{8}$$

where $D(t)$ is the intensity of the disease at time t and the constant $s > 0$ measures the rate

[4]With parental cares $n(t) \neq D(t)$, the indifference condition (4) would instead write

$$k\widetilde{\theta}(t) + q_L(t) - p_L - D(t) = n(t) - D(t)\,.$$

at which the disease intensifies; parameter $v > 0$ measures instead the effectiveness of health care. Linear models are standard in the eradication literature (see Anderson and May, 1991, and Barret and Hoel, 2008, *inter alia*), and are now commonly adopted to investigate the evolution of several diseases, among which breast cancer (see Wellings and Jensen, 1973, Nowell, 1976, Reya *et al.*,2001, *inter alia*).[5]

The maximum problem of hospital i can be formulated as follows:

$$\max_{q_i(t)} \int_0^\infty \pi_i(t)\, e^{-\rho t} dt$$

subject to the state equation (8) and the initial condition $D_0 = D(0) > 0$. The discount rate $\rho > 0$ is assumed to be the same for both firms. The relative size of ρ and s will play a key role in shaping our results. To this regard, it seems natural to assume $\rho < s$ in order for hospitals to attach a proper importance to the future of patients, which additionally spills over positively to the present value of the profit flow.[6]

The game is fully non cooperative, with simultaneous play at every t, with two controls (quality levels, one for each player) and a single state (the intensity of the disease). The solution concept is the Markov perfect equilibrium, which assumes that players can observe the evolution of state and their current actions are conditioned on current time and the current state of the world, that is, the whole previous history is summarised in the current state. In such a case, the current course of action of any players depends on the current state of the world.[7] The analysis of the Markov perfect equilibrium of the game is developed in the next section.

3 The Markov perfect equilibrium

The Bellman equation of hospital i is:

$$\rho V_i(D(t)) = \max_{q_i(t)} \left\{ \pi_i(t) + \frac{\partial V_i(D(t))}{\partial D(t)} \dot{D} \right\} \tag{9}$$

[5]For an exhaustive survey, see Bombonati and Sgroi (2010).

[6]This specific point is also relevant in the field of environmental and resource economics, where low discounting increases the welfare of future generations (see, e.g., Stern, 2007; Nordhaus, 2007; and Weitzman, 2007).

[7]This specific rule is also known as "feedback closed-loop".

where $V_i(D(t))$ is firm i's value function. The first order conditions are:[8]

$$\frac{2c\left[\overline{\theta}(1-k)-p_H+p_L+q_H-q_L\right]-(1-k)\left[p_H-v(1-k)\dfrac{\partial V_H(D)}{\partial D}\right]}{(1-k)^2}=0 \tag{10}$$

$$\frac{2c\left[k(p_H-q_H)-D(1-k)-p_L+q_L\right]-k(1-k)\left[p_L-vk(1-k)\dfrac{\partial V_L(D)}{\partial D}\right]}{k^2(1-k)^2}=0 \tag{11}$$

from which we get optimal qualities

$$\begin{aligned} q_H^* &= \frac{p_H-2c\left(\overline{\theta}-p_H-D\right)+kp_L-v(1-k)\left(\dfrac{\partial V_H(D)}{\partial D}+k^2\dfrac{\partial V_L(D)}{\partial D}\right)}{2c}; \\ q_L^* &= \frac{k\left[p_H+p_L-v(1-k)\left(\dfrac{\partial V_H(D)}{\partial D}+k\dfrac{\partial V_L(D)}{\partial D}\right)\right]-2c\left(k\overline{\theta}-p_L-D\right)}{2c}. \end{aligned} \tag{12}$$

Before proceeding any further, observe that plugging the above solutions into (6) one obtains the following expressions:

$$\begin{aligned} x_H^* &= \frac{p_H-\dfrac{\partial V_H(D)}{\partial D}(1-k)v}{2c}; \\ x_L^* &= \frac{p_L-\dfrac{\partial V_L(D)}{\partial D}k(1-k)v}{2c}. \end{aligned} \tag{13}$$

Since the low-quality output is the only possible source of a quadratic term in the entire problem, if $V_L(D)$ is linear the whole game is necessarily linear in D as well, as there is no reason to suppose $V_H(D)$ to be quadratic. With this in mind, we may now turn to the explicit solution of the Bellman equations, that simplify as follows:

$$\begin{gathered} p_H\left(2\frac{\partial V_H(D)}{\partial D}(1+k+2c)v-p_H\right)+ \\ \frac{\partial V_H(D)}{\partial D}\left[4kp_L-(1-k)\left(\frac{\partial V_H(D)}{\partial D}(1+3k)+4\frac{\partial V_L(D)}{\partial D}k^2\right)v\right]v- \\ 4\left[\frac{\partial V_H(D)}{\partial D}\left(sD+\left((1+k)\overline{\theta}-p_L-2D\right)v\right)-\rho V_H(D)\right]c=0 \end{gathered} \tag{14}$$

[8]Henceforth, we will omit the time argument for the sake of brevity.

for firm H, and

$$p_L\left(4\frac{\partial V_H(D)}{\partial D}(k+c)v - p_L\right) +$$

$$\frac{\partial V_L(D)}{\partial D}\left[2p_H(1+k) - (1-k)\left(2\frac{\partial V_H(D)}{\partial D}(1+k) + \frac{\partial V_L(D)}{\partial D}(3+k)k^2\right)v\right]v -$$

$$4\left[\frac{\partial V_L(D)}{\partial D}\left(sD + \left((1+k)\overline{\theta} - p_H - 2D\right)v\right) - \rho V_L(D)\right]c = 0 \tag{15}$$

for L. To reach a closed-form fully analytical solution, we have to conjecture the explicit form of the value function $V_i(D)$. On the basis of the above considerations, we pose that $V_i(D)$ is linear in D:

$$V_i(D) = \varepsilon_i D + \phi_i,\ i = H, L. \tag{16}$$

Using the above functions, we may simplify the Bellman equations so as to obtain the following system:

$$4Dc(\rho - s + 2v)\varepsilon_H = 0 \tag{17}$$

$$p_H[2\varepsilon_H(1+k+2c)v - p_H] + 4c\rho\phi_H -$$

$$\varepsilon_H\left[4c\left(\overline{\theta}(1+k) - p_L\right) - 4kp_L + v\left(\varepsilon_H(1+3k) + 4\varepsilon_L k^2\right)(1-k)\right]v = 0 \tag{18}$$

for hospital H, and

$$4Dc(\rho - s + 2v)\varepsilon_L = 0 \tag{19}$$

$$p_L[4\varepsilon_L(k+c)v - p_L] + 4c\rho\phi_L + \varepsilon_L[p_H(1+k) -$$

$$2c\left(\overline{\theta}(1+k) - p_H\right) - v\left(2\varepsilon_H(1+k) + \varepsilon_L(3+k)k^2\right)(1-k)\Big]v = 0 \tag{20}$$

for L. Hence, it appears that our conjecture concerning the linearity of the value functions was indeed correct.[9] The above equations are to be solved w.r.t. the unknown parameters appearing in the firms' value functions $\{\varepsilon_H, \varepsilon_L, \phi_H, \phi_L\}$.

From (17) and (19), one immediately obtains $\varepsilon_H = \varepsilon_L = 0$, which means that the two firms' value functions are indeed constants, i.e., $V_i(D) = \phi_i$. Then, solving (18) and (20), we have:

$$\phi_H = \frac{p_H^2}{4c\rho}\,;\ \phi_L = \frac{p_L^2}{4c\rho}\,. \tag{21}$$

[9] We have also checked the alternative possibility where the value function of i is $V_i(D) = \gamma_i D^2 + \varepsilon_i D + \phi_i$, finding that $\gamma_i = 0$, so that the linear form obtains.

Of course, the expressions appearing in (21) measure the discounted profit flows, as the perfect Markov equilibrium profits accruing to firm i are $\pi_i^M = \rho\phi_i$.

There remains to impose stationarity on the state dynamics, whereby $\dot{D} = 0$ delivers:

$$D^M = \frac{\left[p_H(1+k) + 2p_L k - 2c\left(\overline{\theta}(1+k) - p_H - p_L\right)\right]v}{2(s-2v)c}. \tag{22}$$

From (12), the equilibrium qualities are:

$$\begin{aligned} q_H^M &= \frac{p_H(s-(1-k)v) + sp_L k - 2c\left[\left(\overline{\theta} - p_H\right)s + \left(p_H - p_L - \overline{\theta}(1-k)\right)v\right]}{2(s-2v)c} \\ q_L^M &= \frac{2csp_L + s\left(p_H + p_L - 2\overline{\theta}c\right)k + \left[p_H(1-k) + 2c\left(p_H - p_L - \overline{\theta}(1-k)\right)v\right]}{2(s-2v)c} \end{aligned} \tag{23}$$

while from (6) we get the equilibrium output levels $x_i^M = p_i/(2c)$, $i = H, L$, respectively. Hence, $X^M = x_H^M + x_L^M = (p_H + p_L)/(2c)$.

From the discussion above, we can state

Proposition 1 *For any given price vector $\{p_H, p_L\}$, the game yields a unique Markov perfect equilibrium.*

It is worth noting that at the steady state, for a given price vector, (i) full market coverage does not obtain in general, and (ii) the sub-population of patients being treated becomes chronic, i.e., as D^M is constant but positive. We'll come back to these issues in the next section.

Before proceeding in that direction, an obvious complement to Proposition 1 consists in assessing the effects of variations in prices on the equilibrium quality levels q_i^M. To this regard, there exists a lively debate in the literature, with contrasting results both on the theoretical side and on the empirical one (see, e.g., Gravelle and Masiero, 2000; Brekke *et al.*, 2007; and Karlsson, 2007, for theoretical discussions, and Kessler and McClennan, 2000; Tay, 2003; and Gowrisankaran and Town, 2003, for empirical findings). The relevant partial derivatives are

$$\begin{aligned} \frac{\partial q_H^M}{\partial p_H} &= \frac{(1+2c)s - (1+2c-k)v}{2(s-2v)c} \\ \frac{\partial q_L^M}{\partial p_L} &= \frac{(2c+k)s - 2cv}{2(s-2v)c} \end{aligned} \tag{24}$$

implying a non-monotonicity result that we may formulate in the following corollary.

Corollary 2 *Define:*

$$s_1 \equiv 2v,\ s_2 \equiv \frac{(1+2c-k)\,v}{1+2c},\ s_3 \equiv \frac{2cv}{2c+k},$$

where $s_1 > s_2 > s_3$*. For all* $s > s_1$, $\partial q_i^M/\partial p_i > 0$, $i = H, L$*. The same holds for all* $s \in (0, s_3)$*. Inside this range,*

1. *for all* $s \in (s_2, s_1)$, $\partial q_i^M/\partial p_i < 0$, $i = H, L$*;*

2. *for all* $s \in (s_3, s_2)$, $\partial q_H^M/\partial p_H > 0$ *while* $\partial q_L^M/\partial p_L < 0$*.*

Figure 1: Health care levels and prices

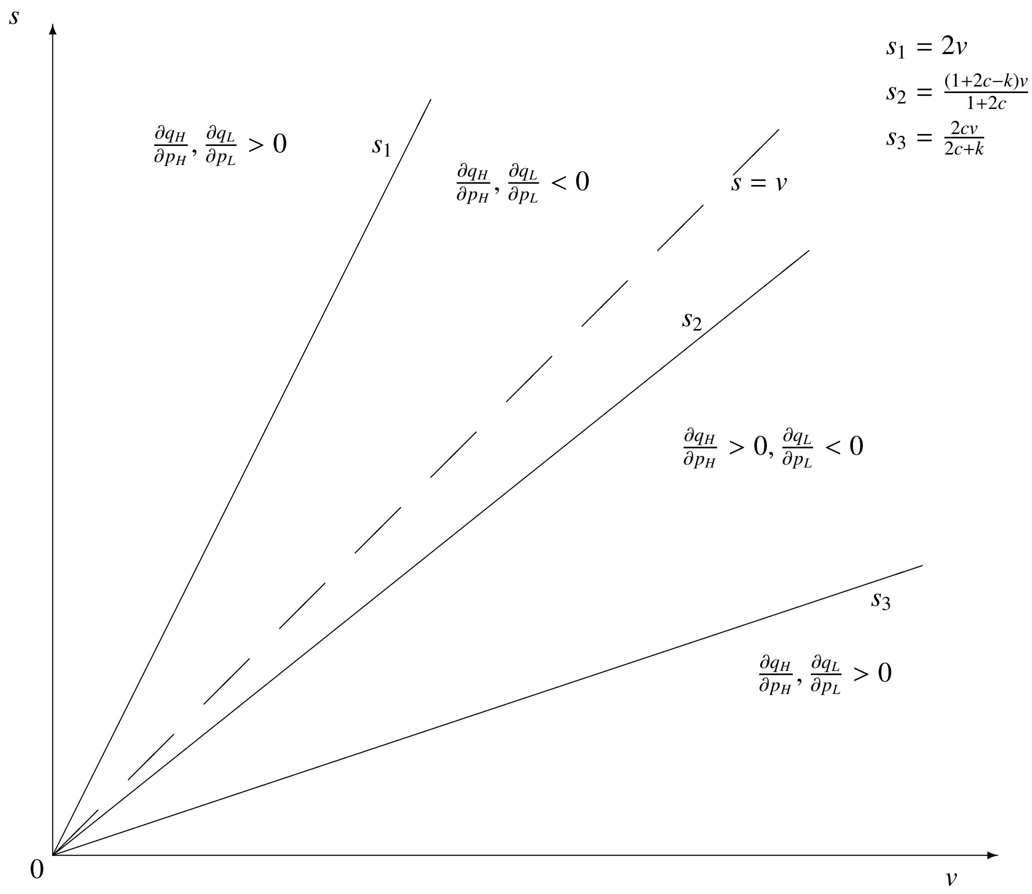

Figure 1 shows the relationship between health care levels and prices. The dashed line represents $s = v$**.** Corollary 2 shows that, in quite extreme regions where the growth rate of the disease is either much higher or much lower than the marginal effectiveness of either medical care, each quality increases in its own price, all else equal. In the intermediate range, as s

decreases, first we observe a negative effect of price increases on quality levels, and then a further switch in the opposite direction. In particular, along $s = v$, $\partial q_i^M / \partial p_i$ is negative for both firms. The results of Corollary 2 can be explained as follows. Hospital profits are driven by two channels: (i) the mark-up from high-income patients (mark-up effect) and (ii) the demand size (demand effect). These two effects are alternative. By raising the mark-up, the demand size decreases and vice versa.

An increase in prices raises the mark-up and decreases demand size. If profits are mainly determined by the mark-up effect, then hospitals have an incentive in raising the quality of health care. Conversely, if profits are mainly driven by the demand effect, then hospitals choose a lower health care levels in order to compensate the fall in demand.

When the marginal effectiveness of health care is not very effective against the growth rate of the disease ($s > s_1$), then health care has a great value, which entails a high mark-up effect. When the marginal effectiveness of health care and the growth rate of the disease are similar ($s \in (s_2, s_1)$), then a lower mark-up can be extracted for a single cure, implying a stronger demand effect. When the marginal effectiveness of health care is mildly higher than the growth rate of the disease ($s \in (s_3, s_2)$), then the incentives of high and low-quality hospitals differ. The high-quality hospital has a stronger mark-up effect, and raises its health care quality, whereas the low-quality hospital lowers its quality to increase the demand size. When the marginal effectiveness of health care is very effective against the growth rate of the disease ($s < s_3$), then the demand is relatively low, so that hospital profits are mainly pushed by the mark-up effect.

Having characterised the optimal behaviour of the two firms and the reaction of optimal qualities to any price changes, we are now in a position to investigate the objectives of the public agency in charge of regulating prices, and the consequent design of the related measures.

4 Regulating prices

In this section we investigate the analysis price regulation of the problem considered. We begin by showing that a welfare-maximising approach to price regulation is not effective. The steady-state social welfare SW^M is given by the sum of profits and consumer surplus (which contains the disease):

$$SW^M = \int_{\widehat{\theta}}^{\overline{\theta}} (z + q_H(t) - p_H - D(t))dz + \int_{\widetilde{\theta}}^{\widehat{\theta}} (kg + q_L^M - p_L - D(t))dg + \pi_H^M + \pi_L^M,$$

where the values of each component are

$$\begin{aligned}
\int_{\widehat{\theta}}^{\overline{\theta}} (z + q_H(t) - p_H - D(t))dz &= \frac{p_H (p_H + 2kp_L)}{8c}, \\
\int_{\widetilde{\theta}}^{\widehat{\theta}} (kg + q_L^M - p_L - D(t))dg &= \frac{kp_L^2}{8c^2}, \\
\pi_H^M &= \frac{p_H^2}{4c},\ \pi_L^M = \frac{p_L^2}{4c}.
\end{aligned}$$

Maximising SW^M with respect to p_H and p_L, respectively, yields:

$$\frac{\partial SW^M}{\partial p_H} = \frac{p_H (1 + 2c) + kp_L}{4c^2} > 0,$$

$$\frac{\partial SW^M}{\partial p_L} = \frac{p_L (k + 2c) + kp_H}{4c^2} > 0.$$

Since these derivatives are always positive, social welfare maximisation is not achievable through prices manipulation.

Given the results of welfare maximisation and the nature of the problem at hand, we can argue that the regulator should manipulate prices so as to achieve a twofold objective, namely, to ensure that (i) the entire population has access to medical care (i.e., what we usually define as full market coverage), and (ii) the disease be completely healed, at least at the steady state equilibrium. We are about to show that there exists an admissible range for $\overline{\theta}$, wherein price regulation attains both objectives at the same time. This translates into solving the system

$$\begin{aligned} X^M - 1 &= 0 \\ D^M &= 0 \end{aligned} \tag{25}$$

w.r.t. p_H and p_L. This yields the unique pair:

$$\begin{aligned} p_H^R &= \frac{2c\left[\overline{\theta}(1 + k) - 2(c + k)\right]}{1 - k} \\ p_L^R &= \frac{2c\left[2c - \left(\overline{\theta} - 1\right)(1 + k)\right]}{1 - k} \end{aligned} \tag{26}$$

with

$$
\begin{aligned}
p_H^R &> 0 \,\forall\, \overline{\theta} > \frac{2(c+k)}{1+k} \\
p_L^R &> 0 \,\forall\, \overline{\theta} < \frac{2c+1+k}{1+k} \\
p_H^R &> p_L^R \,\forall\, \overline{\theta} > \frac{4c+1+3k}{2(1+k)}
\end{aligned}
\tag{27}
$$

which also entail $x_i^M > 0$ and $x_H^M > x_L^M$. Moreover, it must also be true that $q_H^M > q_L^M$ in order to exclude the arising of a quality leapfrogging problem in correspondence of the regulated price vector. This requires:

$$
\overline{\theta} > \frac{2c(2c+1-k)+(1-k)k}{2c(1+k)+(1-k)k} \tag{28}
$$

Finally, it is easy to verify that

$$
\frac{2c+1+k}{1+k} > \frac{2c(2c+1-k)+(1-k)k}{2c(1+k)+(1-k)k} > \max\left\{1, \frac{4c+1+3k}{2(1+k)}, \frac{2(c+k)}{1+k}\right\}. \tag{29}
$$

The foregoing discussion can be summarised in what follows.

Proposition 3 *For all* $\overline{\theta} \in \left(\frac{2c(2c+1-k)+(1-k)k}{2c(1+k)+(1-k)k}, \frac{2c+1+k}{1+k}\right)$, *prices*

$$
p_H^R = \frac{2c\left[\overline{\theta}(1+k)-2(c+k)\right]}{1-k}\,;\; p_L^R = \frac{2c\left[2c-\left(\overline{\theta}-1\right)(1+k)\right]}{1-k} \tag{30}
$$

ensure the universality of medical cares and the disappearance of the disease in steady state.

Hence, the result stated in the above Proposition conveys the substantive message that, for any $\overline{\theta}$ low enough to ensure that both firms hold positive market shares and sufficiently large to exclude the arising of leapfrogging incentives, there exists a unique pair of regulated prices yielding the universality of medical treatment and the elimination of the disease. This also clarifies why it is reasonable to set the salvage value equal to zero.

5 Concluding remarks

We have modelled the interplay between vertical differentiation and the endogenous evolution of a disease in a dynamic duopoly with heterogenous patients. In addition to the unicity of the Markov perfect equilibrium, our analysis yields a clearcut message as to the design of the price regulation policy, as there exists a parameter range wherein price regulation simultaneously delivers two eggs in one basket, namely, universal service and the elimination of disease.

Our results suggest that the regulating prices policy needs to take into account the relationship between the marginal effectiveness of health care and the growth rate of the disease. When these are similar, an increase in regulating prices might surprisingly decrease the health care quality.

To this regard, it is interesting to stress that a traditional utilitarian approach based on the maximization of the welfare function would not ensure the attainment of either the first or the second objective. This appears indeed to be a case in which the standard utilitarian view is not tuned with the nature of the problem at hand. This entails that an effective health policy should be focused on solving the specific problems of this market (disease eradication, full health coverage), rather than accounting for welfare maximisation.

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