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Energy Input Interaction in US Production

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This paper estimates production functions with factor interaction for annual US output from 1949 to 2013 including energy Btu input with capital and labor. Interactions between the three factors are parsimoniously introduced to the error correction estimates. The findings are as follows: Fixed capital assets successfully control for technological change. Interaction between capital and energy reveal them to be very weak substitutes or complements. Factor price elasticities involving the labor force are strong. Labor is overpaid by 39% relative to its declining productivity, while energy is underpaid by 16% relative to its increasing productivity. The own wage effect is nearly elastic implying wage increases are met with nearly opposite percentage decreases in cost minimizing labor input.

Keywords: energy input; factor interaction; factor productivity; factor price elasticities

JEL Classifications: D24; Q41

Estimates of factor price substitution are critical for economic theory, applications, and model simulations. The present estimates of the US aggregate production function treat total energy Btu input as a primary factor of production along with fixed capital assets and the labor force in annual data from 1949 to 2013. Including energy input improves the estimates as in Thompson (2016). The functional form adds interaction terms to the familiar log linear production function in parsimonious fashion. Natural logs of the series are difference stationary leading to robust error correction estimates. The direct estimates allow tests of constant returns to scale and competitive factor markets as null hypotheses.

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The lack of unexplained output trends in the error correction estimates suggests fixed capital assets must imbed evolving technology over the seven decades. Strong positive capital-energy interaction reveals a downward trending output elasticity of labor. In sharp contrast, the output elasticity of energy has a pronounced upward trend. Relative to productivity, labor is overpaid by 16% and capital by 18% at the expense of energy input.

Factor price elasticities derived from the estimates differ substantially from the typical weak or moderate factor price substitutes reported in the literature. Capital and energy are very weak substitutes perhaps to the point of complements. The elastic own energy substitution suggests rising energy prices do not raise spending on energy. The own wage effect is also nearly elastic. Some discussion of the policy implications of these results are included in the Conclusion.

Section 1 reviews the empirical literature on energy substitution. Section 2 reviews the theory of factor price elasticities focused on factor interactions. Section 3 examines properties of the present series. Section 4 presents the estimates starting with log linear production and parsimoniously adding second order interaction terms. Section 5 derives the implied factor price elasticities followed by a section comparing trends in derived output elasticities and factor shares.

1 A Brief Review of the Literature on Energy Substitution

Estimates of energy price elasticities came to the forefront during the energy crises of the 1970s and 1980s when the price of oil tripled. Hudson and Jorgenson (1973), Berndt and Wood (1975), and Ehud and Melnik (1981) report that capital and energy are complements while Griffin and Gregory (1976) find weak substitutes. These early studies rely on estimated systems of translog factor share equations as summarized by Hunt (1986) and Apostolakis (1990).

The literature since the energy crisis is split between reports of capital-energy substitutes and complements. Chang (1994), Caloghiro, Mourelatos, and Thompson (1997), Barnett, Reutter, and Thompson (1998), Kemfert (1998), Mahmud (2000), Christopoulos and Tsionas (2002), Urga and Walters (2003), Koetse, de Groot, and Florax (2008), and Ma and Stern (2016) all report capital and energy are substitutes. In contrast, Moroney (1992), Kintis and Panas (1989), Medina and Vega-Cervera (2001), Frondel and Schmidt (2002), Frondel (2004), and Arnberg and Bjørner (2007) report complements. In a review of over fifty articles, Broadstock, Hunt, and Sorrell (2007) report evenly mixed results across different countries, sectors, time periods, specifications, and estimation techniques. Raj and Veall (1997) report that studies on the original Berndt and Wood (1975) data have produced estimates of the capital-energy Allen elasticity that range from -3.9 to 10.8. Sorrell (2014) makes the point that studies on manufacturing that include the input of materials favor capital-energy complements.

Table 1 presents a sample of estimates of factor price elasticities between capital and energy mainly in US, UK, and OECD manufacturing. Estimates are for cost functions except in the last two rows. Capital has a weak to moderate own price elasticity ε_{Kr} that averages -0.31

across these estimates. The energy own price elasticity ε_{Ee} in the second column is somewhat stronger averaging -0.51. Capital and energy range from weak cross price substitutes to very weak complements. Capital is more sensitive to the price of energy as ε_{Ke} ranges from 0.57 to -0.18 with an average of 0.32. Energy input is insensitive to the price of capital with ε_{Er} ranging from -0.16 to 0.17 with an average of 0.07.

Table 1. Estimates of Factor Price Elasticities

		ε_{Kr}	ε_{Ee}	ε_{Ke}	ε_{Er}
Hudson-Jorgenson (1974)	US, 1947-71	-0.42	0.07	-0.02	-0.18
Berndt-Wood (1975)	US, 1947-71	-0.44	-0.49	-0.16	-0.17
Griffin-Gregory (1976)	US, OECD, 1955-69	-0.18	-0.79	0.13	0.15
Berndt-Khaled (1979)	US, 1947-71	-0.37	-0.45	-0.10	-0.13
Anderson (1981)	US, 1971	-0.02	-0.25	-0.10	-0.09
Pindyck (1979)	OECD GDP, 1971	-0.44	-0.75	0.02	0.28
Hunt (1986)	UK, 1960-80	-0.49	-0.28	0.17	0.21
Christopoulos-Tsionas(2002)	Greece, 1970-1990	-0.44	-0.85	0.06	0.36
Frondel (2004)	US, 1947-1971			-0.17	-0.18
Thompson (2013)	US GDP, 1970-2007	-0.64	-0.60	0.07	0.27
Thompson (2016)	US GDP, 1951-2008	-0.56	-0.72	0.30	0.57
Average		-0.31	-0.51	0.32	0.07

2 Factor Interaction and Factor Price Elasticities

The theory of factor price elasticities is based on cost minimizing behavior in the foundations of Allen (1938), Ferguson and Pfouts (1962) and Berndt and Christensen (1973) as clearly developed in Takayama (1993). In the present specification, output Y is produced with capital K, labor L, and energy E inputs. The present estimates in differences of natural logs start with log linear production parsimoniously adding interaction terms.

Starting in log levels with all three interaction terms, the estimated production function is:

$$\ln Y = \alpha_0 + \alpha_1 \ln K + \alpha_2 \ln L + \alpha_3 \ln E + \alpha_4 \ln K \ln L + \alpha_5 \ln K \ln E + \alpha_6 \ln L \ln E \qquad (1)$$

The theoretical motivation for interaction is the familiar cross effect on marginal products. For instance, the log marginal effect of energy on output in $\partial \ln Y/\partial \ln E$ is affected by the log of capital input according to $\partial^2 \Delta \ln Y/\partial \ln E \partial \ln K = \alpha_5$. These cross effects improve fit and influence the derived marginal products and factor price elasticities.

The translog production function of Christensen, Jorgensen, and Lau (1973) adds the second order own effects $\ln K^2$, $\ln L^2$, and $\ln E^2$ to (1). Direct estimates of the translog are impossible due to the high collinearity between the first order and second order effects. The translog is estimated in a truncated system of factor share equations with the factor share of capital a residual of output or value added. Estimates of factor share equations require assuming constant returns to scale and competitive factor markets. The present specification adds flexibility to the log linear specification without these assumptions.

Competitive factor markets would imply the log linear marginal elasticites in (1) are factor payment shares of income. Let e, r and w denote the price of energy, return to capital, and the wage rate, respectively. Define ψ_E as $\partial \ln Y/\partial \ln v$ where $v = E, K, L$ and ψ_v represents their respective factor shares. The log partial derivative for energy,

$$\psi_E \equiv \partial \ln Y/\partial \ln E = Y_E E/Y = \alpha_3 + \alpha_5 \ln K + \alpha_6 \ln L \tag{2}$$

would equal the energy factor share $\theta_E \equiv eE/Y$ if marginal product Y_E were equal to price e. Competitive markets for capital and labor with $Y_K = r$ and $Y_L = w$ where r is the return to capital and w the wage would similarly imply $\theta_K = \psi_K$ and $\theta_L = \psi_L$.

The homothetic interactive production function (1) may exhibit constant returns to scale CRS depending on estimated α_i parameters. Euler's theorem for product exhaustion $Y = Y_K K + Y_L L + Y_E E$ would follow from CRS and competitive factor markets implying $\Sigma\psi_i = 1$. Estimates of the translog system of derived factor share equations require these assumptions with the capital share equation dropped to avoid overdetermination as discussed by Thompson (2006).

The log linear production function with its first order effects in (1) implies the constant factor partial output elasticities $\psi_K = \alpha_1$, $\psi_L = \alpha_2$, and $\psi_E = \alpha_3$ implying $\alpha_1 + \alpha_2 + \alpha_3 = 1$ as the sufficient condition for CRS. Competitive factor markets would imply factor shares sum to one. For the present interactive production function (1) sufficient conditions for CRS are the first order condition plus the second order conditions of a zero sum in $\alpha_4 + \alpha_5 = 0$ for K, $\alpha_4 + \alpha_6 = 0$ for L, and $\alpha_5 + \alpha_6 = 0$ for E. These conditions are tested as null hypotheses in the present estimates.

Factor price elasticities are derived from the Lagrangian function for the minimization of constrained cost Γ facing factor prices r, w, and e,

$$\Gamma = rK + wL + eE + \lambda[Y - f(.)] \tag{3}$$

where f(.) is the production function. The Lagrangian multiplier λ is the marginal cost of output $\partial\Gamma/\partial Y = \Gamma_Y \equiv \lambda$. Marginal cost has unit value $\Gamma_Y = 1$ in the present model with GDP output.

First order conditions for the constrained cost minimization (3) are:

$$\Gamma_\lambda \equiv \partial\Gamma/\partial\lambda = Y - \mathrm{f}(.) = 0 \quad (4)$$

$$\Gamma_K = r - \lambda f_K,\ \Gamma_L = w - \lambda f_L,\ \Gamma_E = e - \lambda f_E.$$

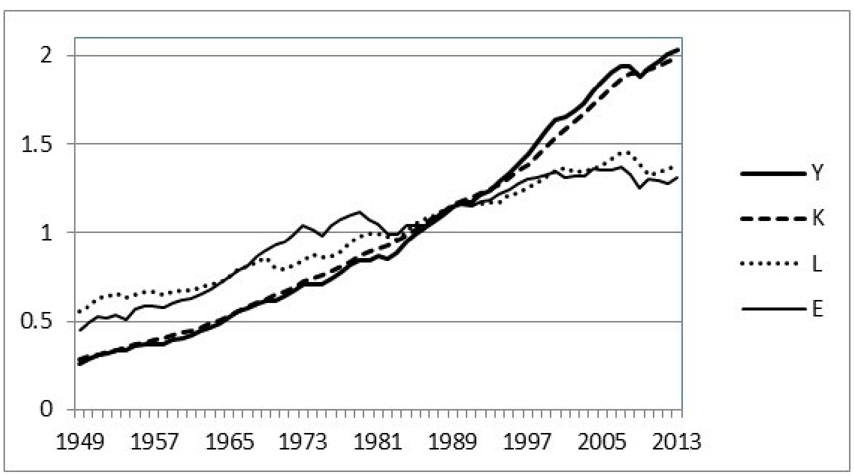
Marginal products f_i are derived as $f_K \equiv \partial f/\partial K = \psi_K K^{-1}$, $f_L = \psi_L L^{-1}$, and $f_E = \psi_E E^{-1}$.

The Hessian matrix is built from the first order conditions in (4). Cross effects in the Hessian include second order effects such as $f_{EE} = -\psi_E E^{-2}$ and $f_{KE} = f_{EK} = \alpha_5(KE)^{-1}$. Factor price elasticities are derived from partial derivatives of the inverted Hessian. For instance, the elasticity of capital relative to the price of energy is $\varepsilon_{Ke} \equiv eK_e/K = f_E K_e/K$ where $K_e \equiv \partial K/\partial e$. Derivation of factor price elasticities assumes competitive factor markets, $\lambda = \Gamma_Y = 1 = rK/\psi_K K = wL/\psi_L = eE/\psi_E$ from (4).

3 The Present Series and Factor Intensities

Figure 1 plots the annual mean weighted series over the years 1949-2013. Gross domestic product Y and fixed capital assets K are indices from the Bureau of Economic Analysis (2015). The fixed capital asset series K includes consumer durables with the aim of the broadest production model. The full-time equivalent labor force L from the Bureau of Labor Statistics (2015) comes from two historical series averaged for the overlapping years 1968 and 1969. Energy E is gross primary Btu input for the entire economy from the Energy Information Administration (2015). Energy consumed in housing and transportation is included to capture all activity in the economy.

Figure 1. Trends in Output Y, Capital K, Labor L, and Energy E



Gross domestic product Y in Figure 1 grows at an average rate of 3.20% with standard deviation $\sigma = 2.33\%$. Output growth diminishes over time at the estimated rate $d\ln Y/dt = -0.042\%$. Fixed capital assets K grow slower at 3.02% and much more steadily with σ = 0.92% at the slightly slower decreasing rate $d\ln K/dt$ = -0.037%.

Labor L grows with occasional decreases before slowing in the late 1990s with the digital revolution and again following the 2008 financial crisis. Over the sample, labor grows at an average of 1.41% at the decreasing rate $d\ln L/dt$ = -0.026% with very high 2.51% standard deviation.

Energy input E has the most varied history with growth until the energy crises then slowing with labor at the digital revolution and financial crisis, structural breaks verified in the estimates. Energy grows at an average 1.65% with the highest 3.19% standard deviation and the fastest decreasing rate $d\ln E/dt$ = -0.064%. Capital provides the foundation for output growth with labor and energy providing variation.

Table 2 reports augmented Dickey-Fuller (1979) tests indicating the series are difference stationary except for strong persistence in the capital series and mild heteroskedasticity for energy. The series prove cointegrated in the production function estimates with robust error correction effects across specifications.

Table 2. Difference Stationary ADF Tests

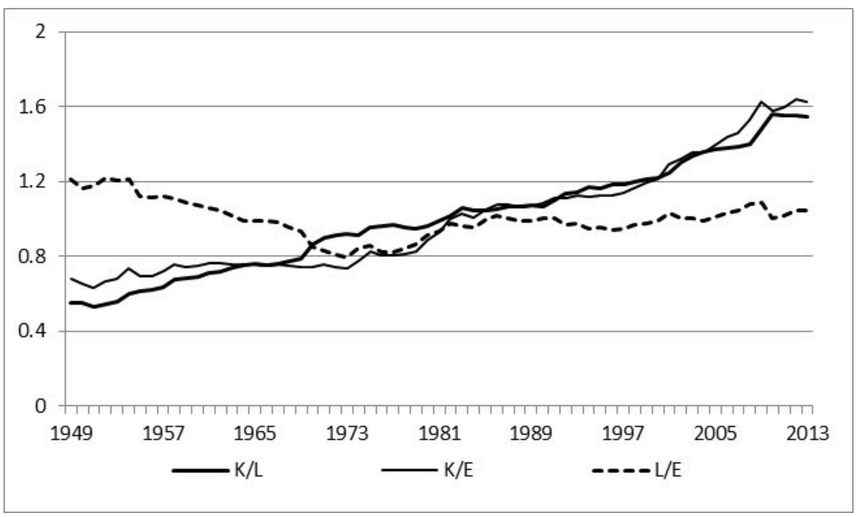
	ADF(1)	ADF(2)
$\ln Y$	-1.20	-0.71
F	2.88	1.31
DW	2.02	1.99
ARCH(1)	0.33	0.23
$\ln K$	-0.48	-0.06
F	81.3^{**}	63.8^{**}
DW	1.57	2.07
ARCH(1)	0.25	0.97
$\ln L$	-2.59	-1.71
F	4.74	2.63
DW	1.91	1.96
ARCH(1)	0.07	0.47
$\ln E$	-0.92	-0.58
F	2.88	1.69
DW	2.01	1.91
ARCH(1)	3.08^{*}	3.31^{*}

**1% *5% $t_c = -3.45$ $F_c = 4.88$

Structural breaks at the energy crises, the onset of the digital age in the 1990s, and the financial crisis of 2008 are familiar in the literature and supported in each of the present series by Perron (1989) structural break tests. A preliminary estimate of the production function with only capital and labor exhibits strong evidence of a structural break at the energy crisis. Including energy input relieves the evidence for this energy crisis structural break. These structural breaks have minimal impacts on the estimated coefficients in the production functions and are not included in the reported estimates.

Figure 2 plots factor intensity ratios. The increasing ratio *K/L* is consistent with a rising *w/r* in growth theory, and the increasing *K/E* with a rising e/r due to energy resource depletion. The factor ratio *L/E* declines up to the energy crisis when it changes direction suggesting e/w began to rise due to resource scarcity. Overall, these factor intensity patterns suggest rising w and falling r with e steady before the energy crisis but rising afterward.

Figure 2. Trends in Factor Intensity Ratios



4 Estimated Interactive Production Functions

The estimated error correction models start with the log linear specification and add second order interaction terms. Each interaction term is considered on its own followed by the three different pairs and all three together.

A significant intercept or constant term in the difference equation estimates would imply an unexplained trend in output. A positive constant would be taken as evidence of improving technology that requiring a different specification to derive factor price elasticities. A negative constant term would imply an unexplained downward trend in output suggesting problems with model specification, omitted variables, or data.

Sufficient conditions for CRS include the first order condition FOC based on the Cobb-Douglas production function $a_1 + a_2 + a_3 = 1$ in (1) with the second order conditions SOC $a_4 + a_5 = 0$ for *K*, $a_4 + a_6 = 0$ for *L*, and $a_5 + a_6 = 0$ for *E* implying $a_4 = a_5 = a_6 = 0$. The present estimates test these FOC and SOC as null hypotheses.

Table 3 reports error correction estimates for the log linear LogLin model and the three specifications *KL, KE*, and *LE* with those single interaction terms. The series are cointegrated according to the Engle-Granger EG (1987) statistics in the cointegrating regressions leading to the consistently robust error correction EC effects.

Table 3. Log Linear and Single Interaction Terms

	Log*L*in	*KL*	*KE*	*LE*
a_0	-1.00^{**}	0.58	0.06	7.55^{**}
	(0.33)	(0.47)	(0.46)	(2.78)
K	0.63^{**}	-0.96	-0.46	0.80^{**}
	(0.20)	(0.74)	(0.52)	(0.20)
L	0.43^{**}	-0.18	0.39^{**}	-1.27
	(0.08)	(0.28)	(0.07)	(0.89)
E	0.43^{**}	0.54^{**}	0.05	-1.23
	(0.05)	(0.05)	(0.18)	(0.91)
KL		0.16^{*}		
		(0.07)		
KE			0.12^{**}	
			(0.05)	
LE				0.16^{*}
				(0.08)
EC	-0.21^{**}	-0.34^{**}	-0.33^{**}	-0.30^{**}
AIC	-6.23	-6.28	-6.34	-6.30
DW	1.68^{g}	1.59^{g}	1.62^{g}	1.65^{g}
ARCH(1)	0.35	1.53	1.39	1.44
FOC	1.49^{**}	-0.60	-0.02	-1.70
	(0.22)	(0.80)	(0.55)	(1.29)
SOC		0.16^{*}	0.12^{**}	0.16^{*}
		(0.07)	(0.05)	(0.08)
ψ_K	0.63^{**}	0.85^{**}	0.85^{**}	0.80^{**}
	(0.20)	(0.05)	(0.04)	(0.20)
ψ_L	0.43^{*}	0.43^{**}	0.39^{**}	0.48^{**}
	(0.08)	(0.09)	(0.07)	(0.05)
ψ_E	0.43^{**}	0.54^{**}	0.51^{**}	0.58^{**}
	(0.05)	(0.05)	(0.07)	(0.05)
$\Sigma\psi$	1.49^{**}	1.82^{**}	1.75^{**}	1.86^{**}
	(0.22)	(0.12)	(0.11)	(0.21)

**1% *5%

Residual correlation is not an issue in any of the estimates according to the Durbin-Watson (1951) DW critical grey area values $D_L = 1.41$ and $D_U = 1.77$. Residual heteroskedasticity is not indicated in any of the estimates according to the reported autoregressive residual ARCH(1) tests.

The reported total effects in Table 3 are derived from the coefficient in differences in the error correction estimate minus the product of the negative error correction EC effect and the corresponding coefficient from the cointegrating regression. Estimates including any second order own effect introduce multicollinearity and are not selected by the Aikake (1973) Information Criterion AIC. Estimates including energy input are consistently selected by the AIC over preliminary estimates with only capital and labor. Estimates including structural breaks are not selected by the AIC.

The log linear Log*L*in model in the first column of Table 3 has a significant error correction coefficient EC and significant input coefficients. The negative intercept term implies an unexplained downward trend in output. The high energy coefficient ψ_E rejects sufficient conditions for CRS. An estimate in differences does not reject CRS with its sum of ψ_i elasticities equal to 1.02 (0.22).

The interaction terms in Table 3 reduce the linear coefficients and increase the derived factor output elasticities ψ_I in the Log*L*in estimate. The significant positive constant with *LE* interaction implies an unexplained positive output trend. The *LE* interaction term is destructive to the estimate of the Log*L* in model. The FOC and SOC for CRS are consistently rejected. The sums of the implied factor elasticities $\Sigma\psi_i > 1$ reject product exhaustion in the Euler theorem.

Table 4 reports estimates with multiple interaction terms with the three pairs of input interaction *KL-KE*, *KL-LE*, and *KE-LE* and the specification with all three interaction terms. Five of the nine interaction terms across these models are significant. Interaction between *K* and *E* noticeably increases explanatory power. The *KE-LE* model would be selected by the AIC although its negative constant term implies an unexplained negative output trend. Both the *KL-KE* and *KE-LE* models reject the FOC for CRS. Four of the twelve models reject the SOC for CRS. The Euler theorem of product exhaustion is rejected in every specification as $\Sigma\psi_i \neq 1$. In the *KL-KE* model for instance, $\Sigma\psi_i = 1.05$ on average over the sample.

Table 5 summarizes tests of null hypotheses across specifications. The TECH column reports whether the null hypothesis that the intercept term equals zero is rejected. The four specifications that do not reject in the TECH column are consistent with constant technology and the derived factor price elasticities.

Table 4. Multiple Interaction Terms

	KL-KE	*KL-LE*	*KE-LE*	*KL-KE-LE*
a_0	0.69 (0.48)	-13.97* (6.94)	-6.95 (7.47)	-13.6 (8.33)
K	-1.17 (0.80)	-2.59* (1.54)	-2.54* (1.37)	-5.11* (2.78)
L	-0.14 (0.42)	1.59 (1.43)	4.25* (2.24)	5.08* (2.86)
E	0.48 (0.35)	2.85 (1.91)	3.29* (1.86)	5.84* (2.73)
KL	0.16 (0.12)	0.31* (0.15)		0.34* (0.15)
KE	0.02 (0.09)		0.32* (0.14)	0.21 (0.14)
LE		-0.20 (0.17)	-0.36* (0.20)	-0.54* (0.29)
EC	-0.33**	-0.36**	-0.32**	-0.34**
AIC	-6.19	-6.18	-6.34	-6.18
DW	1.58g	1.52g	1.64g	1.52g
ARCH(1)	1.07	1.03	0.53	0.45
FOC	-0.83 (0.97)	1.85 (2.84)	5.00 (3.22)	5.81 (4.83)
SOC_K	0.17 (0.13)	0.31* (0.15)	0.32* (0.14)	0.55** (0.20)
SOC_L	0.16 (0.12)	-0.11 (0.22)	-0.36* (0.20)	-0.20 (0.32)
SOC_E	0.02 (0.09)	-0.20 (0.17)	-0.04 (0.24)	-0.33 (0.32)
ψ_K	0.86** (0.05)	0.91** (0.09)	0.96** (0.10)	1.02** (0.16)
ψ_L	0.47** (0.09)	0.58** (0.12)	0.32** (0.12)	0.47** (0.05)
ψ_E	0.56** (0.01)	0.59** (0.06)	0.44** (0.09)	0.54** (0.04)
$\Sigma\psi$	1.89** (0.11)	2.08** (0.15)	1.72** (0.17)	2.03** (0.19)

**1% *5%

The FOC and SOC columns in Table 5 report tests of the first and second order sufficient conditions for CRS. All specifications reject either the FOC or SOC. Selection based solely on the AIC would be either the *KE* or the *KE-LE* models providing evidence that capital-energy interaction improves the estimates.

Table 5. Specification Summary

	TECH	FOC	SOC	AIC
Log*L*in	reject	reject		-6.23
KE		reject	reject	-6.34
LE	reject	reject	reject	-6.30
KL		reject	reject	-6.28
KL-KE		reject		-6.19
KL-LE	reject			-6.18
KE-LE		reject	reject	-6.34
KL-KE-LE	reject		reject	-6.18

5 Derived Factor Price Elasticities

Table 6 reports the derived factor price elasticities for the models in Table 5 that have no unexplained output trends with the linear model LogLin included for comparison. Factor price elasticities are evaluated at sample means by standard errors derived from error propagation calculations with standard errors of coefficients and standard deviations of variables. The sum of elasticities for each factor across factor prices equals zero along a given isoquant with factors paid marginal products. The Cournot condition that the share weighted sum of price elasticities for each factor price holds does not hold in the unrestricted estimates.

Capital, labor, and energy are inelastic substitutes with the null hypothesis of a zero cross price elasticity not rejected in half of the estimates. Cross price elasticities of capital K are typically weaker than for labor *L* and energy *E*. Classifying elasticities above 0.50 as strong, the only strong cross price elasticities are ε*L*r with *KE* interaction, ε*Ew* with *KL-KE* interaction, and ε*Le* in the two models with *KL-KE* interaction. Own price elasticities for capital are weak in four of the six models. Own price elasticities for L and E are consistently strong and elastic or nearly so including *KE* interaction.

Comparing these results to the literature in Table 1 suggests KE interaction strengthens factor price elasticities. In the *KL-KE* model the own energy elasticity *εEe* less than -1 implies an increase in the price of energy would lower the energy share of income. The own labor elasticity ε*Lw* is also nearly less than -1 in the *KL-KE* model implying an increased wage would have no noticeable effect on labor income.

Positive interaction between two inputs weakens the cross-price elasticity between those two inputs. For instance, the positive *KE* interaction in Table 3 leads to the weaker insignificant corresponding cross price elasticities in Table 6. An increase in the price of energy lowering its cost minimizing input would lower the marginal product of capital making it less attractive as a substitute. This weakening effect is apparent in the *KL* interaction model.

Table 6. *KLE* Factor Price Elasticities

KL	*r*	*w*	*e*	*KE*	*r*	*w*	*e*
K	-0.46**	0.16	0.30*	K	-0.44**	0.22*	0.23
	(0.18)	(0.13)	(0.16)		(0.16)	(0.12)	(0.16)
L	0.31	-0.66*	0.35**	L	0.50*	-0.82**	0.31**
	(0.25)	(0.29)	(0.10)		(0.27)	(0.32)	(0.09)
E	0.48*	0.28**	-0.79**	E	0.40	0.23**	-0.63**
	(0.26)	(0.10)	(0.29)		(0.27)	(0.07	(0.28)
KL-KE	*r*	*w*	*e*	*KE-LE*	*r*	*w*	*e*
K	-0.46	0.21	0.25	K	-0.47**	0.18	0.29*
	(0.37)	(0.30)	(0.31)		(0.18)	(0.14)	(0.16)
L	0.34	-0.99*	0.65*	L	0.31	-0.64*	0.33**
	(0.49)	(0.51)	(0.34)		(0.24)	(0.27)	(0.10)
E	0.40	0.62*	-1.02*	E	0.45	0.30*	-0.75**
	(0.49)	(0.35)	(0.52)		(0.25)	(0.11)	(0.27)
KL-KE-LE	*r*	*w*	*e*	LogLin	*r*	*w*	*e*
K	-0.70*	0.27	0.43	K	-0.58*	0.29	0.29
	(0.30)	(0.33)	(0.37)		(0.31)	(0.24)	(0.24)
L	0.23	-0.73**	0.50*	L	0.42	-0.71*	0.29*
	(0.28)	(0.19)	(0.25)		(0.37)	(0.42)	(0.14)
E	0.27	0.37*	-0.63*	E	0.42	0.29*	-0.71*
	(0.23)	(0.18)	(0.26)		(0.37)	(0.14)	(0.42)

**1% *5%

The negative *LE* interaction terms in the *KL-KE* and *KL-KE-LE* models of Table 4 lead to the stronger cross price elasticities in Table 6. An increase in the price of energy e lowering energy input would raise the marginal product of labor making it more attractive as a substitute. Negative factor interaction does not receive much attention in the literature but may influence adjustments in the economy.

Summarizing results, capital has a moderate own price elasticity that is strongest in the *KL-KE-LE* model with all interaction terms. Changes in the price of capital generate very little if any substitution with labor or energy. Labor has a strong own wage elasticity, nearly elastic in the *KL-KE* model. An increased wage would not much raise labor income with moderate substitution toward energy and little if any effect on cost minimizing capital input. Energy input has a strong own price elasticity, elastic in the *KL-KE* model implying an increase in the price of energy would lower its share of income. An increase in the price of energy creates consistently moderate substitution toward labor with little if any substitution toward capital.

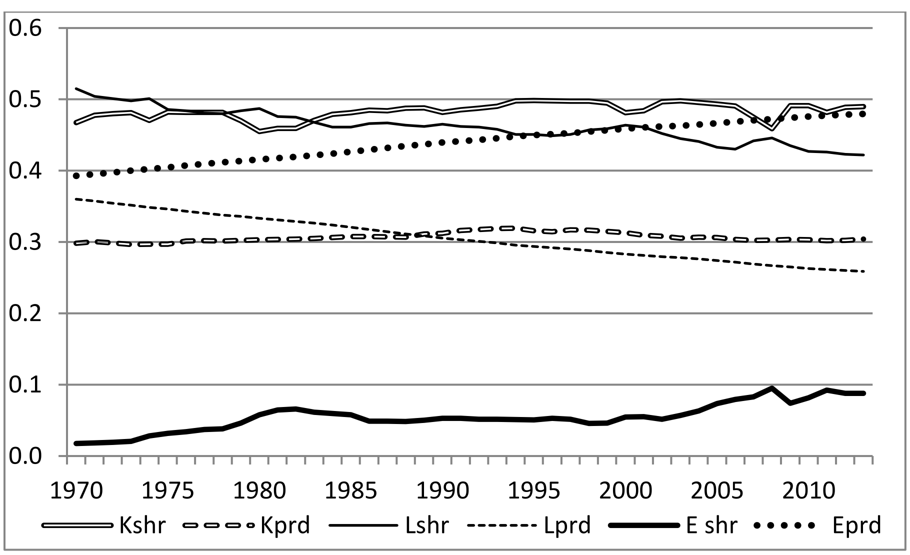
6 Factor Shares and Derived Output Elasticities

Figure 3 compares the factor shares θ_i from the data with the estimated output elasticities ψ_i for the *KL-KE* model in (2). All interaction models in Tables 3 and 4 have very similar output elasticities ψ_i. Figure 3 starts in 1970 with availability of data for labor compensation and energy payment. The capital share θ_K is derived as the residual after labor and energy shares.

Capital is consistently overpaid with its stable output elasticity below its steady factor share by an average of 18% of income. The declining labor share does not keep pace with its declining output elasticity revealing labor as overpaid by 16% over the sample. The upward trending output elasticity of energy is 39% above its slowly rising income share on average. The sum of productivities averages a consistent 1.05 over the sample rejecting Euler's theorem in the estimates. The estimates reveal labor as increasingly overpaid at the expense of increasingly underpaid energy and consistently underpaid capital.

The divergence of productivities and factor shares in Figure 3 is supported by recent models of noncompetitive pricing in factor markets. Juselius (2003) indirectly estimates behavioral equations that lead to noncompetitive pricing. Pintus (2006) explores consistent general assumptions on preferences and technology. Raurich, Sala, and Sorolla (2012) develop a model of imperfect competition in factor markets. The present results suggest extending these capital-labor models to include energy input would provide evidence on the structure of the underlying factor markets.

Figure 3. Trends in Factor Shares and Output Elasticities



7 Conclusion

The present estimates reveal that positive interaction between capital and energy increases explanatory power and weakens substitution between the two inputs to the point of complements. Other factor price elasticities become stronger due to capital-energy interaction. Overpaid labor has declining productivity in contrast to underpaid energy with rising productivity. Overpaid capital has stable productivity. These properties have broad policy implications.

An increase in the capital return would have only minimal impact on gross domestic product due to the moderate own capital price elasticity, weak cross price effect on energy input, and strong substitution toward labor. Policies aiming to control interest rates are misplaced. The artificially low federal funds rate dampens labor demand. The financial crisis of 2008 would have been readily absorbed with strong substitution toward labor.

The strong own wage elasticities averaging -0.77 explain why the legislated minimum wage and union labor agreements have little or no effects on labor income. A more fundamental approach to sagging wages would be to reverse the downward trend in labor productivity that can be blamed on restrictive hiring laws, job tenure, and the National Labor Relations Board.

Controlling the international price of oil has been a major goal of US foreign policy over the past century. The monopsony power of franchised monopoly utilities also contributes to underpricing energy input. Rising energy prices pose little policy concern due to the strong own price elasticity and cross price substitution with labor.

In closing, some implications of the present results across fields in economics can be mentioned. In international trade theory, a tariff on energy imports would not affect import spending. A tariff on labor intensive imports raising the wage would lead to strong substitution toward energy input with little effect on the labor share of income. In resource economics, depletion and rising energy prices leads to strong substitution away from energy input toward labor. In macroeconomics, monetary policy lowering the interest rate lowers the cost minimizing input of labor. In growth theory, investment raises the price of energy instead of the wage.

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