COVID-19 effects on the Canadian term structure of interest rates

FEDERICO SEVERINO[*][†]

Université Laval

MARZIA A. CREMONA[‡]

Université Laval; CHU de Québec - Université Laval Research Center

ÉRIC DADIÉ

Université Laval

In Canada, COVID-19 pandemic triggered exceptional monetary policy interventions by the central bank, which in March 2020 made multiple unscheduled cuts to its target rate. In this paper we assess the extent to which Bank of Canada interventions affected the determinants of the yield curve. In particular, we apply Functional Principal Component Analysis to the term structure of interest rates. We find that, during the pandemic, the long-run dependence of level and slope components of the yield curve is unchanged with respect to previous months, although the shape of the mean yield curve completely changed after target rate cuts. Bank of Canada was effective in lowering the whole yield curve and correcting the inverted hump of previous months, but it was not able to reduce the exposure to already existing long-run risks.

Keywords: Canadian yield curve, COVID-19, monetary policy, Functional Principal Components Analysis

JEL Classifications: E43, E58, G01

1 Introduction and motivation

On March 11, 2020, the director of the World Health Organization (WHO), Dr. T.A. Ghebreyesus, declared: *"We have therefore made the assessment that COVID-19 can be characterized as*

a pandemic. [...] All countries must strike a fine balance between protecting health, minimizing economic and social disruption, and respecting human rights." (WHO Director-General's opening remarks at the media briefing on COVID-19, March 11, 2020).

This address officially marked the start of a health crisis that was already wreaking havoc on the global economy and financial markets. At the social level, on September 28, 2020 the world reached the terrible milestone of one million deaths due to COVID-19, and this death toll has kept growing since then: over 6 million as of May 1, 2022; data from WHO (May 4, 2022) COVID-19 Weekly Epidemiological Update. At the economic level, global economy saw an unprecedented collapse due to the pandemic. The Canadian gross domestic product fell by 7.1% and 11.4% during the periods from February to March 2020 and from March to April 2020, respectively (Statistics Canada, Table 36-10-0434-01). No sector of the economy has been spared from the crisis. The COVID-19 outbreak rapidly increased financial market volatility and augmented investors' fear. As to the Canadian bond market, these phenomena induced a widening of credit spreads, a liquidity shrinkage and a fall of bond funds value (Ouellet Leblanc and Shotlander 2020).

Canada constitutes the world's ninth-largest economy: in 2019 its gross domestic product amounted to $1.74 trillion in current USD, according to The World Bank. In the same year, its general government gross debt amounted to 86.82% of the gross domestic product, according to the IMF. However, an analysis of the term structure of Canadian sovereign bonds during the pandemic is missing in the literature (see the relevant bibliography in Section 2).

In response to COVID outbreak, the Bank of Canada announced several measures to reduce panic and calm down the markets. The intervention with the most powerful impact on bond yields was the huge cut of the overnight rate target (the target rate) from 1.75% to 0.25% in March 2020. This interest rate, which is employed by depository institutions in the overnight market, is the lowest possible interest rate and it has an extremely short-term nature. The central bank introduced several other measures to reduce the financial distress and promote the resilience of the Canadian economy, such as the Contingent Term Repo Facility, the Commercial Paper Purchase Program and the Corporate Bond Purchase Program.[1]

The yield curve (or term structure of interest rates) represents government-bond yields as a function of their time to maturity (or term). It is apparent that the monetary policy intervention of the Bank of Canada affects short-term yields. However, the impact on long-term yields is not straightforward. On the one hand, the mechanic compounding of interest rates makes the change in the target rate propagating across time horizons. On the other hand, the economic literature acknowledges that the long-term side of the term structure is generally mainly associated with expectations and anticipation of market participants about future macroeconomic scenarios. Detailed discussions of the expectation theory and its alternatives can be retrieved in Russell (1992) and in the introductions of Cox, Ingersoll and Ross (1985) and Severino (2022).

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[1]Details on the Bank of Canada website.

Our goal is to assess the extent to which the Bank of Canada interventions affected the determinants of the term structure of bond yields during the first wave of COVID-19 pandemic. We tackle the issue of studying the behavior of the yield curve across different time periods using a data set of Canadian government bond yields ranging from May 1, 2018, to October 30, 2020. We analyze these data employing a nonparametric approach within the framework of Functional Data Analysis (FDA). This allows us to quantify the evolution of the main factors of the yield curve over time, and to relate the changes in such factors to both market uncertainty and monetary policy interventions. Specifically, we provide a comparative study on three sub-periods of the data set (the latter including COVID-19 outbreak). We first employ depth-based functional boxplots to visualize the curve distribution in each period. Then, we apply Functional Principal Component Analysis (FPCA) to each sample of yield curves in order to elicit the components explaining the most variability in each period, similarly to Feng and Qian (2018).

The FPCA of yield curves extends the more common application of the classic (non - functional) Principal Component Analysis (PCA) to yield to maturity (Litterman and Scheinkman 1991) to the functional setting. The aim is to decompose the yields at different horizons into three factors: *level*, *slope* and *curvature*. Similarly to Litterman and Scheinkman (1991), we find three components reflecting the modes of variation of yields in the short-, medium- or long-term, that are not directly observable in the average or standard deviation yield curves. An important advantage of using FPCA instead of PCA is that the time dependence and the ordering of terms in the yield curve are implicitly taken into account by the functional approach, while PCA treats the variables independently of their order. In addition, the interpretation of the functional components is neater, due to the smoothness of the component eigenfunctions, as well as of the means and the covariance functions.

Our analysis is complemented by the estimation of Nelson and Siegel (1987) exponential regression model for the yield curve in each day – which allows us to describe the three factors from a different angle. In particular, the curvature effect, which has a tiny weight in the FPCA variance decomposition, is better appreciated in this model.

Our analyses show that Bank of Canada target rate cuts of March 2020 induced a decline in the entire term structure of yields and imposed a positive monotony – that was absent in the previous twelve months – to the yield curve. Nonetheless, the FPCA reveals the non-negligible presence, during the pandemic, of the same long-run risks detected in the previous months. Central bank interventions had little impact on them. All conclusions are summarized in Section 6.

The paper quantitatively describes the changes in the Canadian yield curve factors induced by COVID-19 outbreak and the timely and massive decisions by the Bank of Canada. In our result interpretations, we do not disentangle the effects of the pandemic from those of the related monetary policy interventions. Indeed, we consider the first wave of COVID-19 and the simultaneous central bank decisions as a unique shock impacting the whole term structure of

interest rates. Moreover, among all the central bank's interventions, we primarily refer to the 150-basis-point target rate cut of March 2020 which constitutes the most disruptive monetary policy measure implemented.

The paper is organized as follows. After the literature review of Section 2, we illustrate the sample, its subdivision in periods and the related descriptive statistics in Section 3. In Section 4 we apply FPCA to the three sample periods. Section 5 completes the analysis by fitting Nelson and Siegel (1987) model. The main findings are, then, elaborated in Section 6. The Appendix contains further analyses and figures.

2 Literature review

Our paper lies at the intersection of different research topics: the determination of yield curve factors, the specificity of the Canadian term structure, the global bond markets during the pandemic, and the application of FDA techniques to financial data. Several papers study the movements in the Canadian term structure of interest rates, but none of them focuses on the period of the first COVID-19 wave, where monetary policy interventions were impressive.

2.1 Level, slope and curvature in the yield curve

Identifying the determinants of the yield curve and forecasting their evolution are major challenges in financial economics. The yield curve reflects the health of economic system and, when sloping downward, is considered a preliminary signal for a financial crisis (Ang, Piazzesi and Wei 2006).

Many studies aim at understanding the main factors of the yield curve. In their seminal work, Litterman and Scheinkman (1991) use PCA to reduce the dimensionality of yields and identify three uncorrelated factors (i.e., *level*, *slope* and *curvature*) explaining more than 98% of U.S. Treasury bonds yields variance: see also Jamshidian and Zhu (1996). The level represents a downward or upward change in interest rates characterized by a parallel shift in the yield curve. The slope (or steepness) reflects a twist caused by long-term rates being higher than short-term rates, or vice versa. Finally, a shock in the curvature generates an increase in both short- and long-term rates, and a simultaneous decrease of intermediate rates, or vice versa (a butterfly). Importantly, the uncorrelation between the three components is crucial for the design of factor neutrality models to immunize portfolios from the movements of such factors: see, e.g., Barber and Copper (1996), Falkenstein and Hanweck (1997), Golub and Tilman (1997). These techniques are largely applied to Asset Liability Management by insurance companies and pension plans, to hedge interest rate risk. See, e.g., Chapter 7 in Veronesi (2016) and the comprehensive exposition in Luckner et al. (2003). Other uses of PCA in the yield curve modelling can be found in Novosyolov and Satchkov (2008) and Barber and Copper (2012); a recent application to European sovereign bonds is provided by O'Sullivan and Papavassiliou (2020).

Nelson and Siegel (1987) suggest an alternative approach to PCA to capture the determinants of the term structure of interest rates. Their exponential regression model, that we recall in eq. (1), approximates the yield curve by using three latent factors, denoted by $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$. Such factors feature long-, short- and medium-term effects and they can be interpreted as *level*, *slope* and *curvature*, respectively (Diebold and Li 2006). Moreover, the literature provides some macroeconomic proxies for such factors. Diebold, Rudebusch and Aruoba (2006) link $\beta_{1,t}$ to a measure of inflation and $\beta_{2,t}$ to demeaned manufacturing capacity utilization. They show that a shock in $\beta_{2,t}$ induces a response in the federal funds rate (set by the central bank), while a shock in $\beta_{1,t}$ increases inflation, capacity utilization and the federal funds rate. Conversely, a shock in the last three variables affects $\beta_{1,t}$ and $\beta_{2,t}$. In addition, Afonso and Martins (2012) analyze the different impacts of fiscal shocks on the three factors $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$.

Numerous other three-factor models for the yield curve, alternative to PCA and the exponential regression approach, have been proposed in the literature: see, e.g., Balduzzi, Das, Foresi and Sundaram (1996), Dai and Singleton (2000). Recently, Ortu, Severino, Tamoni and Tebaldi (2020) use persistence-based factors to model the term structure of interest rates and improve the predictability of bond returns. Another branch of this literature focuses more on identifying observable factors for modelling the yield curve: see, e.g., Evans and Marshall (2007), Ang and Piazzesi (2003), Bikbov and Chernov (2010). Indeed, some macroeconomic variables (inflation, real activity, etc.) largely contribute to explain movements in the yield curve at short, medium or long maturities.

2.2   The Canadian term structure of interest rates

This paper focuses on the changes in the three main determinants of the yield curve (i.e., level, slope and curvature) in the Canadian sovereign bond market. Hence, it adds to the literature on the Canadian term structure of interest rates. In fact, Canadian yield curves share similarities with the U.S. ones. However, as Harvey (1997) point out, the Canadian term structure of interest rates' ability to forecast Canada's economic growth outperforms U.S. interest rate indicators. Cozier and Tkacz (1994) highlight the relation between the slope of the Canadian yield curve and future real income, as well as the one between Canadian term spreads and output. Canadian term spreads turn out to have higher predictive power than U.S. spreads to forecast Canadian output growth, a fact confirmed also by Clinton (1995). Côté and Fillion (1998) describe how the information content of the Canadian term structure is used in monetary policy. Recently, considering the Canadian economy as a whole, Moran, Stevanovic and Touré (2022) explain the impact of COVID-19 outbreak through the lenses of macroeconomic uncertainty shocks.

2.3   COVID-19 and bond markets around the world

Our work focuses on the first wave of COVID-19, with the aim to clarify the effects of the pandemic (and the related expansionary monetary policies) on bond markets. During this period,

the loss of trust in the financial system rapidly led to massive corporate bond sales. As Kargar, Lester, Lindsay, Liu, Weill and Zúñiga (2021) illustrate, the severe illiquidity in U.S. corporate bond markets in March 2020 was mitigated only by several Federal Reserve interventions: the liquidity crisis microstructure is described by O'Hara and Zhou (2021). Similar dynamics are recognized in investment funds in corporate bond markets by Falato, Goldstein and Hortaçsu (2021). Only Federal Reserve announcements were able to calm the disruption in the debt market (Haddad, Moreira and Muir 2021). The severe illiquidity of U.S. Treasury bonds during COVID-19 outbreak, and in particular the wide spreads in March 2020, are also been analyzed by He, Nagel and Song (2022) in a dynamic equilibrium framework. The impact on sovereign credit risk are emphasized by Augustin, Sokolovski, Subrahmanyam and Tomio (2022), who relate a country's sovereign default risk (during the pandemic) to its debt-to-GDP ratio.

In addition, Papailias (2022) provide an analysis of the yield curve before and during the COVID-19 outbreak in the U.S. and Euro area countries. The author exploits Nelson and Siegel (1987) model with a time-varying parameter λ_t to study the persistence of the yield curve factors, documenting a lower degree of persistence (and so more predictability) in the Euro area yield factors. With broader geographic diversification, Zaremba, Kizys, Aharon and Umar (2022) use data from developed and emerging countries to seize the impact of the pandemic on government bond term spreads, while Zaremba, Kizys and Aharon (2021) focus on the effect of policy responses to sovereign bond volatility. Finally, Gubareva (2021) provide an analysis of the liquidity conditions of emerging markets during the pandemic.

2.4 Functional Data Analysis

Our methodology pertains to Functional Data Analysis (FDA), a branch of statistics particularly suited to our purposes since it treats data as smooth curves, allowing one to fully exploit the shape information they comprise. An overview of the most common FDA techniques can be found in Kokoszka and Reimherr (2017). In the realm of time series, some applications of FDA are provided by Cai, Fan and Yao (2000), Ramsay and Ramsey (2002) and Chapter 8 in Fan and Yao (2003). Moreover, Bowsher and Meeks (2008), Kargin and Onatski (2008) and Chaudhuri, Kim and Shin (2016) employ FDA models to forecast the U.S. Treasuries yield curve, the term structure of Eurodollar futures rates and the density of national inflation rates in U.K., respectively. On a larger perspective, an application of FDA to COVID-19 epidemic data can be found in Boschi, Di Iorio, Testa, Cremona and Chiaromonte (2021).

The main tool we employ to analyze the yield curves is Functional Principal Component Analysis (FPCA), a dimension reduction method that permits to extract from a given data set a small number of orthogonal components, which explain the most variance of the data. Such components are smooth curves themselves. See, for instance, Chapters 8 and 9 in Ramsay and Silverman (2005) and Shang (2014). Our FPCA approach on the term structure of interest rates is very similar to that of Feng and Qian (2018). However, they first use B-splines to obtain

smooth yield curves from the raw data, and then perform FPCA on the smooth curves; on the contrary, our PACE approach does not require smoothing of the individual yield curves, but directly incorporates smoothing in the mean and covariance function estimation (details in Subsection 4.1). Feng and Qian (2018) also assess the relevant forecasting power of FPCA with respect to other methods for yield curve prediction. Finally, another example of application of FPCA to financial data (in particular, to volatility) is provided by Müller, Sen and Stadtmüller (2011), while Kneip and Utikal (2001) exploit FPCA to study the evolution of household income and age distribution.

3   Sample subdivision and descriptive analysis

We consider a daily sample of yields from May 1, 2018, to October 30, 2020. The data set can be freely downloaded from the Bank of Canada website at `https://www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/`. For each day, the data set contains the yields to maturity of zero-coupon bonds with term ranging from 3 months to 30 years, on a quarter basis (120 maturities in total).[2] That is, the data set consists of daily yield curves generated from pricing data of Canadian government bonds and treasury bills (see Figure 1). A description of the methodology used to derive the yield curves is provided in Bolder, Metzler and Johnson (2004).

3.1   Sample subdivision

Since we aim at quantifying the effects of the COVID-19 outbreak on the term structure of rates, we partition our data set into three sub-samples, based on the occurrence of important financial events. We observe that the target rate is 1.25 % on May 1, 2018, when our sample begins, and is later modified several times by the Bank of Canada (see Table 1 and top panel of Figure 2).

The first sample split results from the comparison of the 3-month and 10-year yields (see the top and middle panels in Figure 2). The latter is higher at the beginning of the sample, as it is in normal times, but becomes lower later. Hence, we set the beginning of the second period to the first day in which the term spread (the difference between 10-year and 3-month yields) is negative, i.e. to March 22, 2019. The second period is characterized by a negative (or close

Table 1: **Target rates.** Target rates set by the Bank of Canada in the sample period. The rate of May 1, 2018, was fixed previously. Dates in dd/mm/yy format.

Date	01/05/18	10/07/18	24/10/18	04/03/20	16/03/20	27/03/20
Target rate	1.25 %	1.50 %	1.75 %	1.25 %	0.75 %	0.25 %

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[2]If *t* denotes the day of an observation in the sample, the related terms correspond to days *t+(3 months)*, *t+(6 months)*, *t+(9 months)* etc., until *t+(360 months)*.
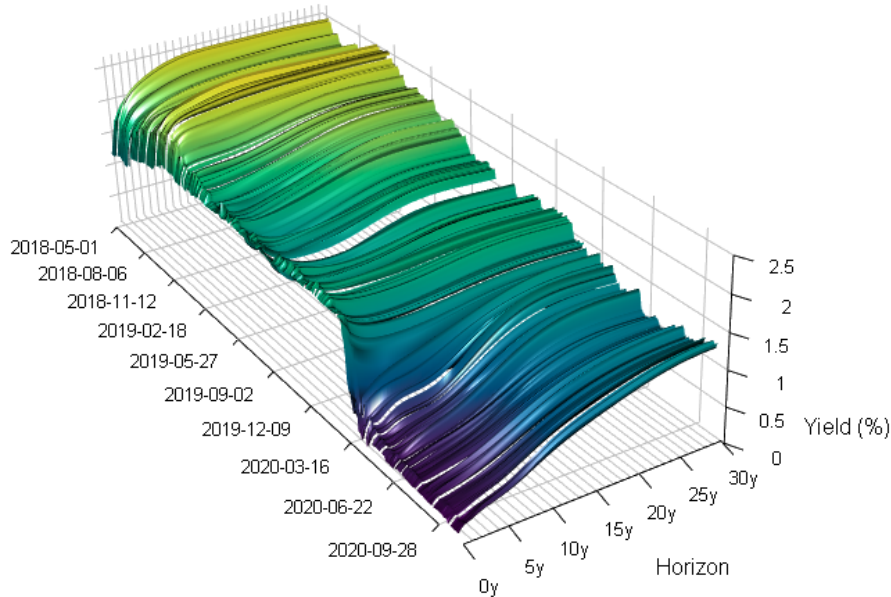
Figure 1: **Daily yield curves from May 1, 2018, to October 30, 2020.** Term structures of Canadian government bond yields in the whole sample period.

to zero) term spread. This quantity is a proxy for the slope of the yield curve (Diebold and Li 2006) and so a change in its sign denotes a curve inversion. The consensus in the literature is that the difference between 10-year and 3-month yields has forecasting power for recessions and output growth (Wheelock and Wohar 2009).

The second period ends before the pandemic outbreak. Although the first rate cut occurs on March 4, 2020, a pre-announcement effect is observable: the 3-month yield remarkably falls on February 28, 2020, and the yield curve features a consistent downward shift. Hence, we choose this date as the cut-off. Our subdivision is supported by functional boxplots based on the 2-curve band depth (Sun and Genton 2011, López-Pintado and Romo 2009). Indeed, including February 28 to March 3, 2020 (a total of three curves) in the second period would have produced three outliers in the corresponding functional boxplot. The selected split excludes the presence of outliers (see Figure A1 in Appendix A1).

We remove from the analysis the period from February 28 to March 27, 2020, due to the exceptionally frequent interventions by the Bank of Canada in cutting the target rate and starting its support to the economy. This phase features a high demand for liquidity in the markets, pervasive uncertainty and financial stress. The yield curves in this removed transition period do not feature a common pattern and largely vary from one day to the other (see Figure A1). Moreover, yield volatility is exceptionally high in this period, especially in the short term (see the bottom panel of Figure 2).
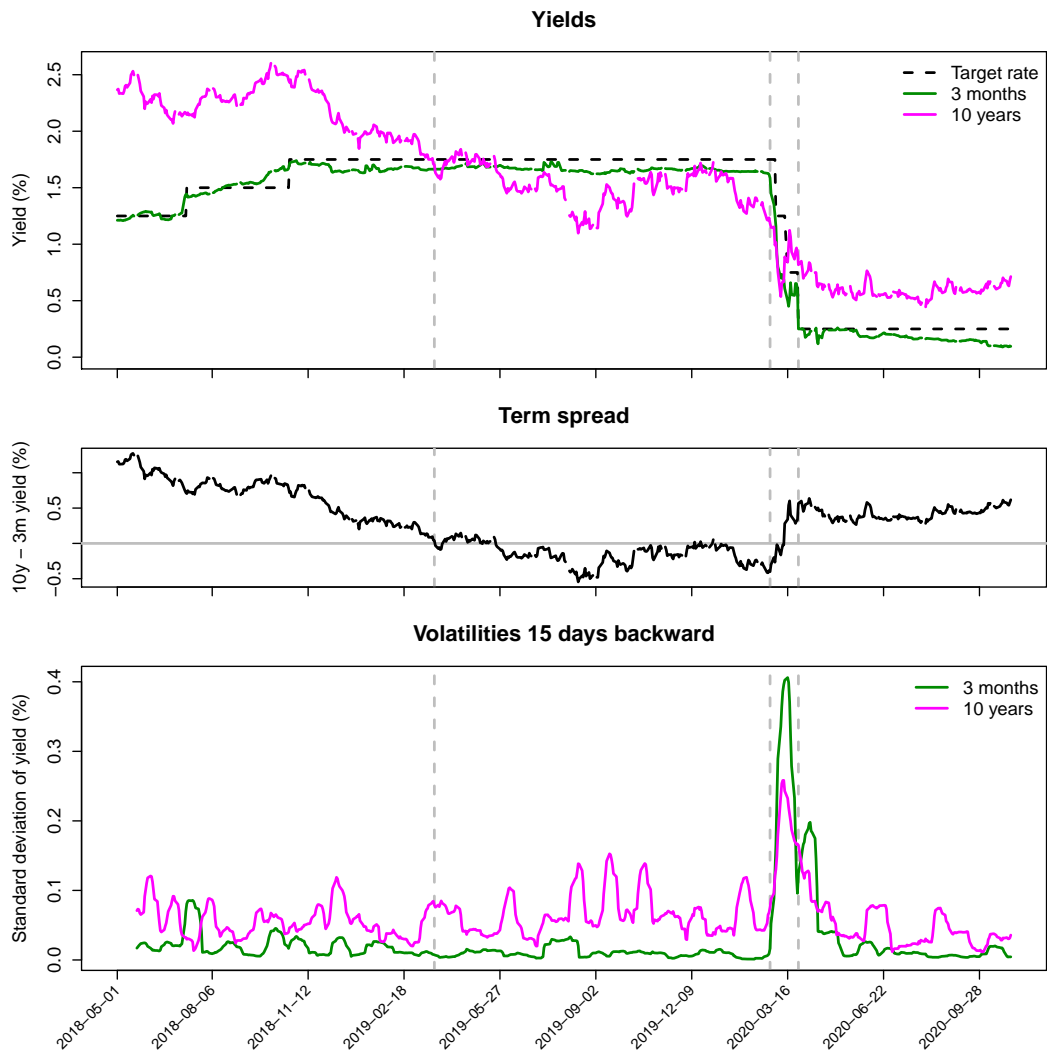
Yields

Figure 2: **Daily yield data from May 1, 2018, to October 30, 2020.** The top panel shows the target rate set by the central bank (black dashed line), the 3-month yield to maturity (green line), and the 10-year yield to maturity (purple line). The middle panel displays the term spread (10-year yield minus 3-month yield). The bottom panel represents the 15-day backward standard deviations of 3-month and 10-year yields (green and purple lines, respectively). The vertical gray dashed lines show our subdivision in periods. All values are in percentages.

From March 28, 2020, no other cuts of target rates take place, the term spread is stable and volatility is under control. This is our third period of analysis, the one related to the first pandemic wave.

As a result, the sub-samples under scrutiny are the following.

- *First period:* from May 1, 2018, to March 21, 2019; 233 observations. The term spread is positive (normal time); the Bank of Canada increases the target rate twice; some tensions are linked to the trade war between China and the United States, but global economic growth keeps solid (in particular in the United States).

- *Second period:* from March 22, 2019, to February 27, 2020; 245 observations. The term spread is negative, on average (anomalous time); the Bank of Canada does not intervene on the target rate; somber economic perspectives anticipated by market actors; slowdown of global economic growth, in particular in China and the Euro area (because of Brexit concerns); trade conflicts between China and the United States are still present.

- *Third period (COVID-19):* from March 28, 2020, to October 30, 2020; 124 observations. The term spread is positive (return to normal time); the Bank of Canada does not intervene on the rates anymore, but renovates its economic support; sharp contraction of the world economy due to COVID-19 first wave (massive reduction of economic activity to limit the virus spread).

More detailed macroeconomic issues are described in the quarterly Monetary Policy Reports by the Bank of Canada (from April 2018 to October 2020).

3.2 Mean and volatility term structure

For each day, we consider the term structure of interest rates, i.e., the curve of bond yields to maturity at different horizons. We then estimate, for each of the three periods considered, the mean yield curve and the relative volatility (coefficient of variation, i.e., standard deviation divided by the mean) curve. Figure 3 shows these curves after smoothing according to the preliminary steps of the FPCA algorithm described in Subsection 4.1. For the sake of completeness, we plot in the same figure the raw curves: pointwise mean and relative volatility. Note that we consider relative volatility in place of volatility because yields variability is naturally lower when yields are low (as in the third period). This problem is overcome by the relative volatility. The figure analogous to Figure 3 for the removed transition period (February 28 to March 26, 2020) is in Appendix A1.

The top panel of Figure 3 shows a neat decline of mean yields (at all horizons) from the first period to the second, and to the third. The decline from the second to the third period follows closely the Bank of Canada target rate cuts in March 2020. Interestingly, the first and the third periods show increasing yield curves, with the first period curve attaining a plateau after few maturities. On the contrary, the average yield curve for the second period features an inverted hump, with medium-term yields lower than short- and long-term ones.

Term structures of means

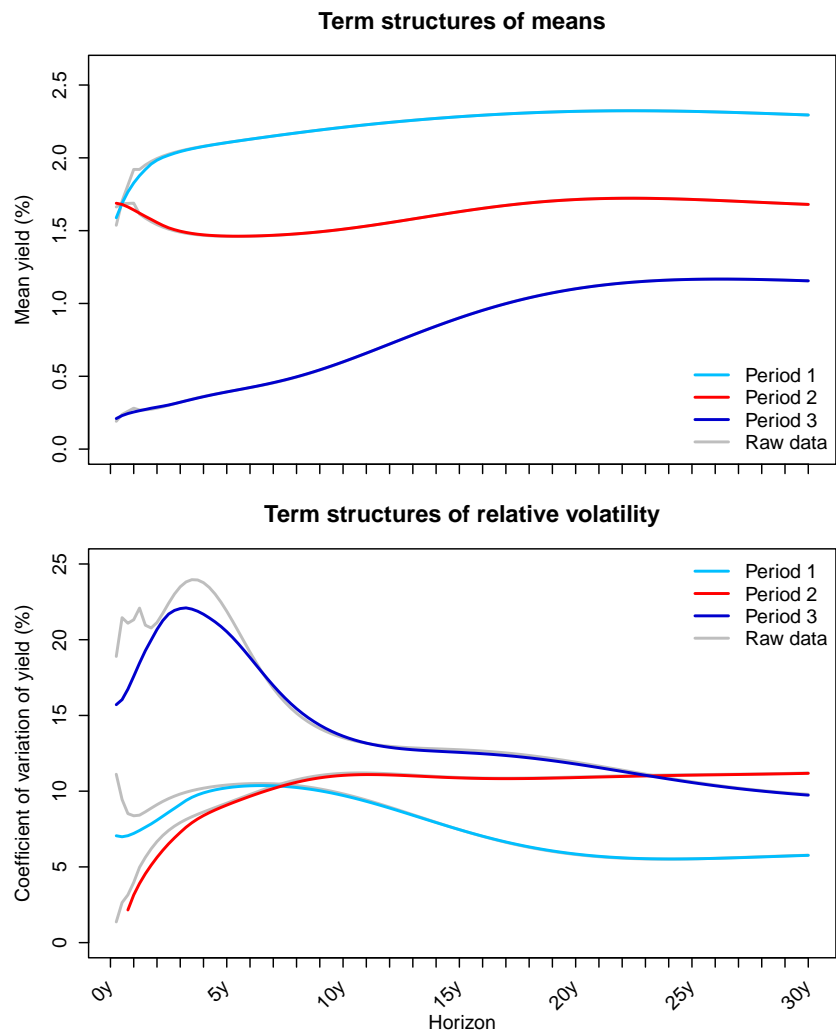**Term structures of relative volatility**

Figure 3: **Mean and relative volatility term structures.** Smoothed term structures of mean (top panel) and relative volatility (bottom panel) of yields to maturity with different horizons, estimated separately for each of the three periods. Light blue, red and blue lines correspond to the first, second and third period, respectively. Grey lines represent the raw curves.

As far as relative volatility is concerned, the third period shows an excess short-term volatility caused by the COVID-19 outbreak. The hump of relative volatility gives an idea of the horizon (roughly 10 years) after which the pandemic consequences on government bonds are expected to completely fade out (the peak is, however, roughly at 3-year maturity). In addition, the relative volatility at long maturities of the COVID-19 period is very similar to the one of the

second period, suggesting the presence of long-run risks with similar magnitude. The central bank interventions do not seem to have affected such sources of uncertainty. Interestingly, the long-run relative volatility is smaller in the first period, where market tensions concentrate in the short and medium run.

To summarize, from mean and relative volatility curves in the three sub-samples, we can infer the following.

- *First period:* increasing mean yield curve with high yields; the curve reaches the plateau quickly; low long-run relative volatility.

- *Second period:* mean yield curve with inverted hump: medium-term yields lower than short- and long-term yields; average yields generally lower than the ones in the first period; sustained long-run relative volatility.

- *Third period (COVID-19):* increasing mean yield curve with low yields; the curve reaches the plateau later than the first period curve; explosion of short-term relative volatility; sustained long-run relative volatility as in the second period.

4 Functional Principal Component Analysis of yield curves

4.1 Methodology

We employ Functional Principal Component Analysis (FPCA) to study the modes of variation of the yield curves in each of the three sub-samples. See, e.g., Chapters 8 and 9 in Ramsay and Silverman (2005). For each period, we consider the yield curves $y_1(x), \ldots, y_N(x)$ observed at horizons $x = 3, 6, \ldots, 360$ as N realizations of the square-integrable stochastic process $Y(x)$, $x \in X = [3; 360]$, with mean $\mu(x) = \mathbb{E}[Y(x)]$ and covariance function $v(x, z) = Cov(Y(x), Y(z))$. We then project the random curve $Y(x)$ into the low dimensional space defined by the first K smooth eigenfunctions $\phi_1(x), \ldots, \phi_K(x)$ of the covariance operator $V : L^2(X) \to L^2(X)$, $V(f) = \int_X v(x, z) f(x) dx$, corresponding to the eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_K$:

$$Y(x) \approx \mu(x) + \sum_{k=1}^{K} \xi_k \phi_k(x),$$

where $\xi_k = \int_X [Y(x) - \mu(x)] \phi_k(x) dx$ are the principal component scores. To perform FPCA and estimate mean, covariance function, eigenfunctions, eigenvalues, and principal component scores, we follow the PCA through Conditional Expectation (PACE) approach of Yao, Müller and Wang (2005) and Liu and Müller (2009), as implemented in the function FPCA of the R package fdapace (Carroll, Gajardo, Chen, Dai, Fan, Hadjipantelis, Han, Ji, Mueller and Wang 2020). In order to incorporate smoothing in the FPCA, we employ local linear smoothing for estimating the mean $\mu(x)$ and the covariance function $v(x, z)$ by setting the argument methodMuCovEst to "smooth". Briefly, the employed PACE approach involves the following steps:

- Compute the estimated mean function $\hat{\mu}(x)$ using local linear smoothing, aggregating the N curves $y_1(x), \ldots, y_N(x)$;

- Compute the estimated covariance function $\hat{v}(x, z)$ by smoothing the sample raw covariance;

- Obtain the estimated eigenfunctions $\hat{\phi}_k$ and eigenvalues $\hat{\lambda}_k$ by performing eigenanalysis on the smoothed covariance function;

- Estimate the principal component scores using numerical integration, i.e. compute $\hat{\xi}_{i,k} = \int_3^{360} [y_i(x) - \hat{\mu}(x)] \hat{\phi}_i(x) dx$.

In each period, we select $K = 3$, obtaining the approximation $y_i(x) \approx \hat{\mu}(x) + \sum_{k=1}^3 \hat{\xi}_{i,k} \hat{\phi}_k(x)$ for each yield curve. This choice leads to a variability explained greater than 99% in all three periods, and matches the choice of three factors (the so-called *level*, *slope* and *curvature*) in PCA on the yield curve usually made in the financial literature (Litterman and Scheinkman 1991).

FPCA results in the three periods are displayed in Figures 4-6, which include barplots of the variance explained by the first three components, the corresponding eigenfunctions, and the shocked yield curves when the shock affects a single component (i.e. the plot of the components as perturbations of the mean).

4.2   Results interpretation

In line with Litterman and Scheinkman (1991), the first three principal components can be interpreted as level, slope and curvature, and explain most of the variations in the yield curves: the variance explained is more than 99% in each period. In particular, the level alone explains always more than 93% of total variance, while the slope has more weight in the second period than in the other two periods.

For each of the three periods, the level (first component) is rather stable across horizons (top-right and bottom-left panels of Figures 4-6). However, the first period features a slightly stronger level in the medium term, a behavior similar to the relative volatility in the same period. In the other periods, the level is higher in the long term than in the short term and a shock to the level induces a vertical shift in the yield curve which is larger in the long term. In fact, the COVID-19 crisis did not meaningfully modify the main mode of variation (the level). Long-run risks are still present and their magnitude is unchanged, even though all yields to maturity are lower in the third period: interest rates continue being persistent.

The first period features a slope (second component) remarkably different from the other periods (top-right and bottom-center panels of Figures 4-6). In particular, in the first period the slope is high (in absolute terms) on maturities less than 5 years, smaller (and of opposite sign) between 5 and 17 years roughly, and becomes zero later. Such behavior is in line with

Variance explained (%)

Functional principal components – Period 1
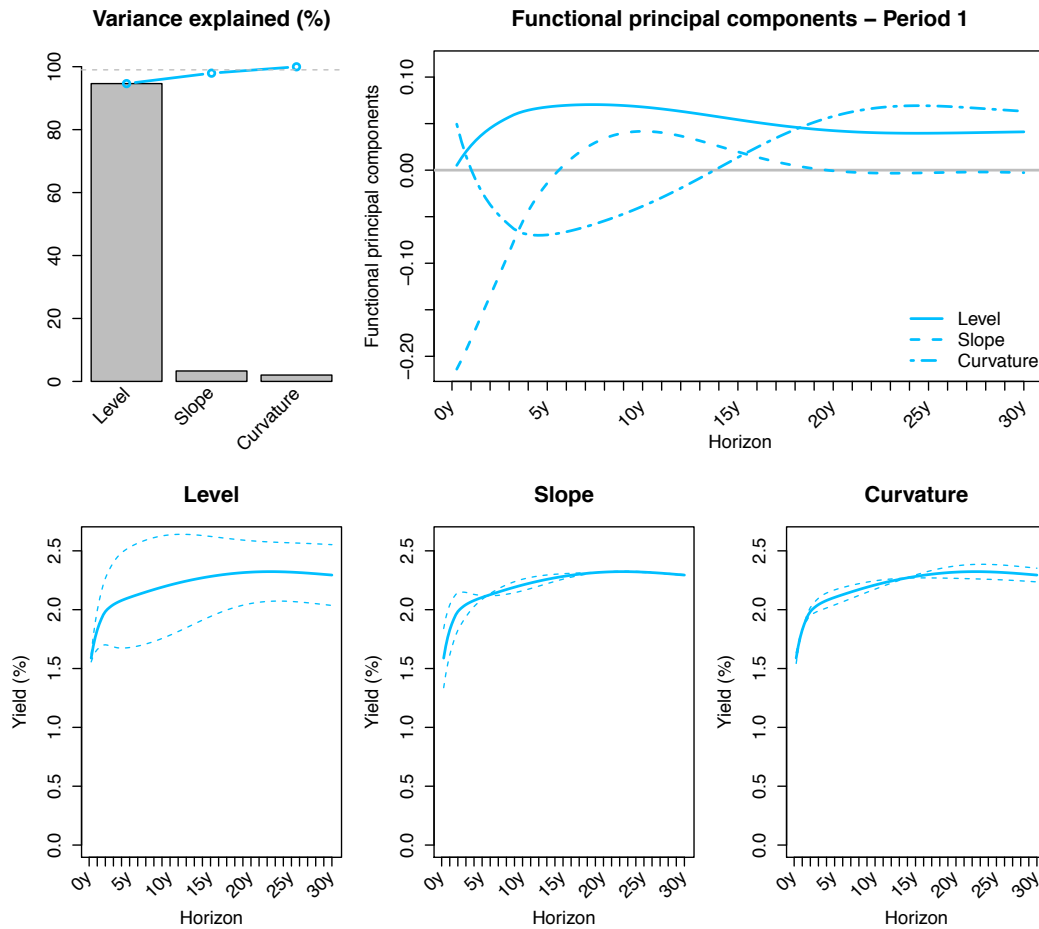
Level

Slope

Curvature

Figure 4: **FPCA results for the first period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenfunctions of such components. The bottom panels represent the (smoothed) mean term structures of yields in the first period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

the pattern of relative variance and the rapid flattening of the mean curve, and it reconfirms the importance of short- and medium-term risks in the first period. A positive shock in the slope can make short-term yield higher than medium-term ones without affecting long-term rates. On the contrary, the slope in the other periods is non-null in the long run (after the 15-year horizon roughly). Interestingly, the slope long-run dependence is qualitatively unchanged during the pandemic. The differences concern the variance explained (6% and 3.22% in the second and third periods, respectively) and the shape of the shocked yield curves. Indeed, in the second

Variance explained (%)

Functional principal components – Period 2

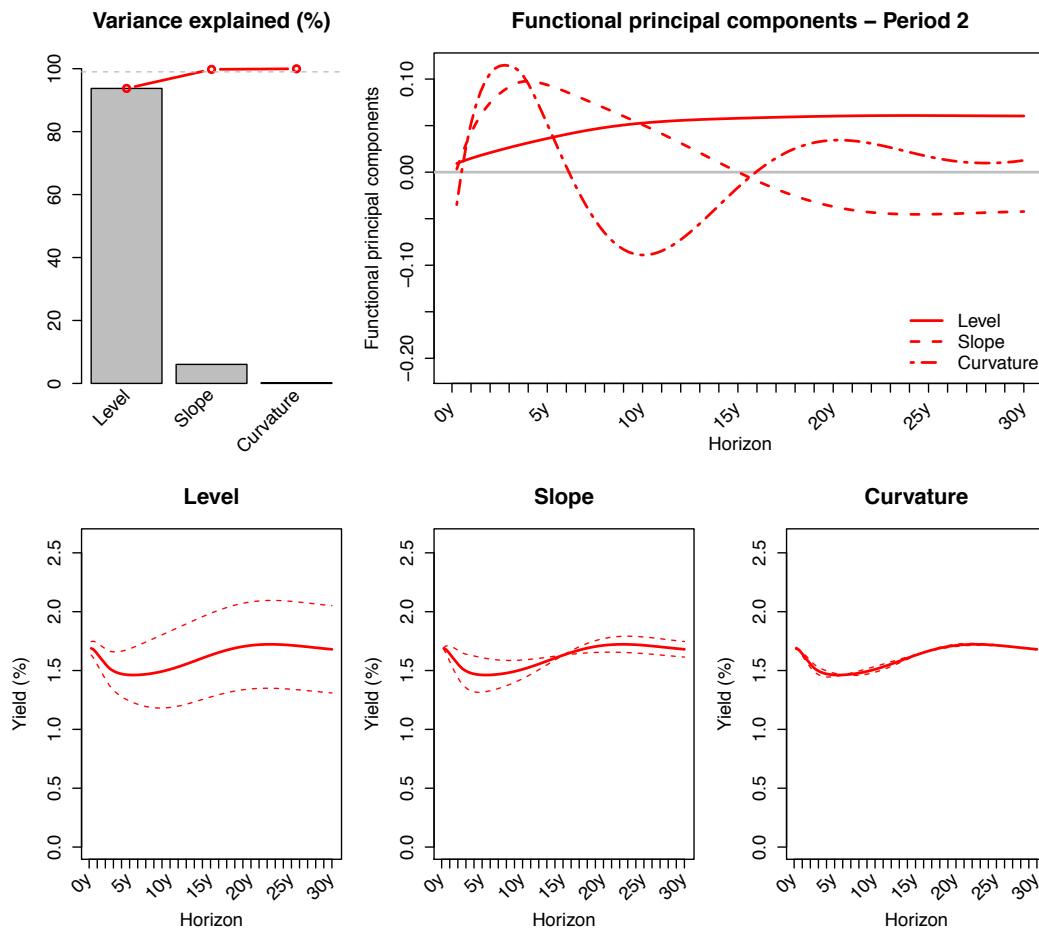**Level**   **Slope**   **Curvature**

Figure 5: **FPCA results for the second period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenfunctions of such components. The bottom panels represent the (smoothed) mean term structures of yields in the second period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

period, a positive shock in the slope can make the yield curve flat or even downward sloping (see the bottom-center plot in Figure 5). In the third period, a modest shock in the slope does not affect the increasing monotony of the yield curve.

In the first two periods, the eigenfunctions of the third principal component (curvature) have the same signs in the short- and in the long-term. The behavior of the curvature is more complex in the third period. The interpretation is more difficult in this case. However, the variance explained by the curvature is tiny in all periods (roughly 2% at most).
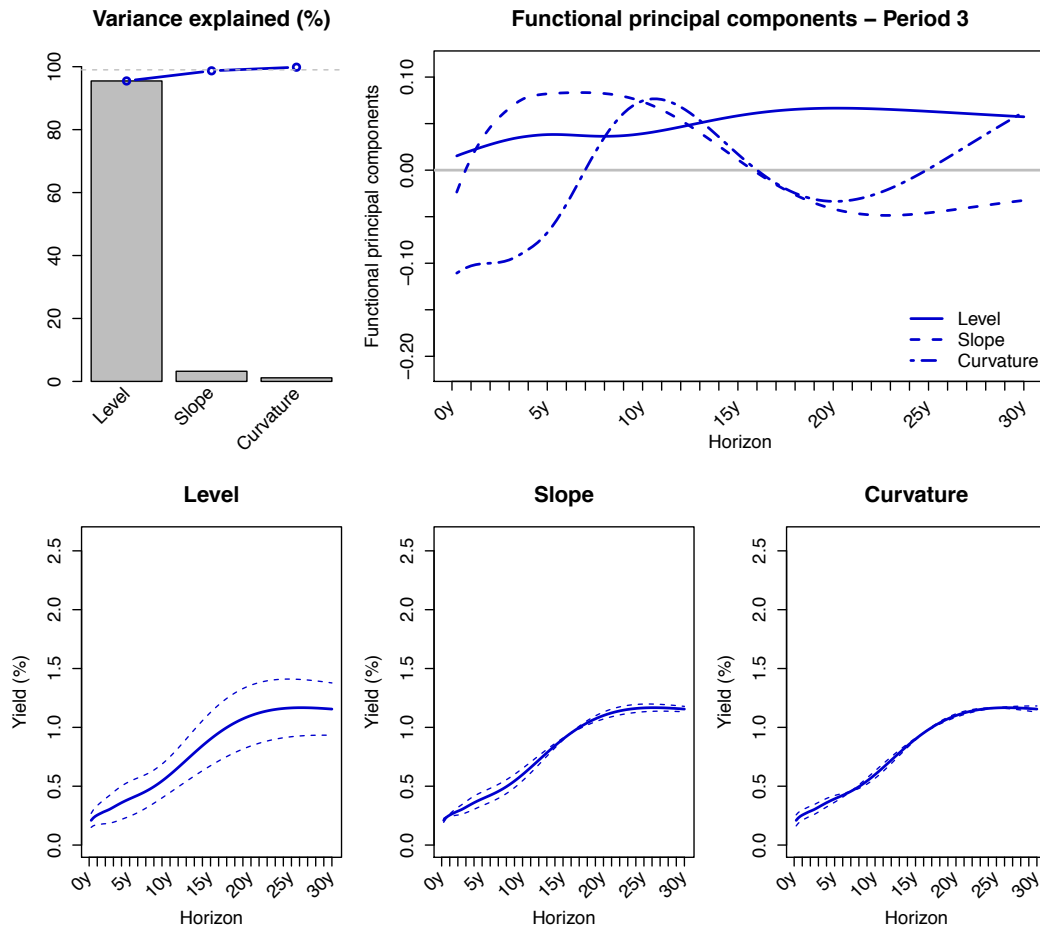
Figure 6: **FPCA results for the third period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenfunctions of such components. The bottom panels represent the (smoothed) mean term structures of yields in the third period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

5 Nelson and Siegel (1987) approach

Nelson and Siegel (1987) model provides an alternative perspective on level, slope and curvature effects. This model allows us to shed some light on the role of the curvature, which does not explain much variability in FPCA, making the interpretation difficult.

5.1 Methodology

Nelson and Siegel (1987) provide a parsimonious exponential approximation of the yield curve,

based on three factors. As illustrated by Diebold and Li (2006), for each day t the yield curve can be estimated via the functional form

$$y_t(x) = \beta_{1,t} + \beta_{2,t}\left(\frac{1 - e^{-\lambda_t x}}{\lambda_t x}\right) + \beta_{3,t}\left(\frac{1 - e^{-\lambda_t x}}{\lambda_t x} - e^{-\lambda_t x}\right), \tag{1}$$

where $y_t(x)$ is the yield to maturity with term x months and λ_t is a positive parameter that governs the exponential decay. Here, $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ are three latent dynamic factors with long-, short- and medium-term effects. As discussed in Diebold and Li (2006), $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$
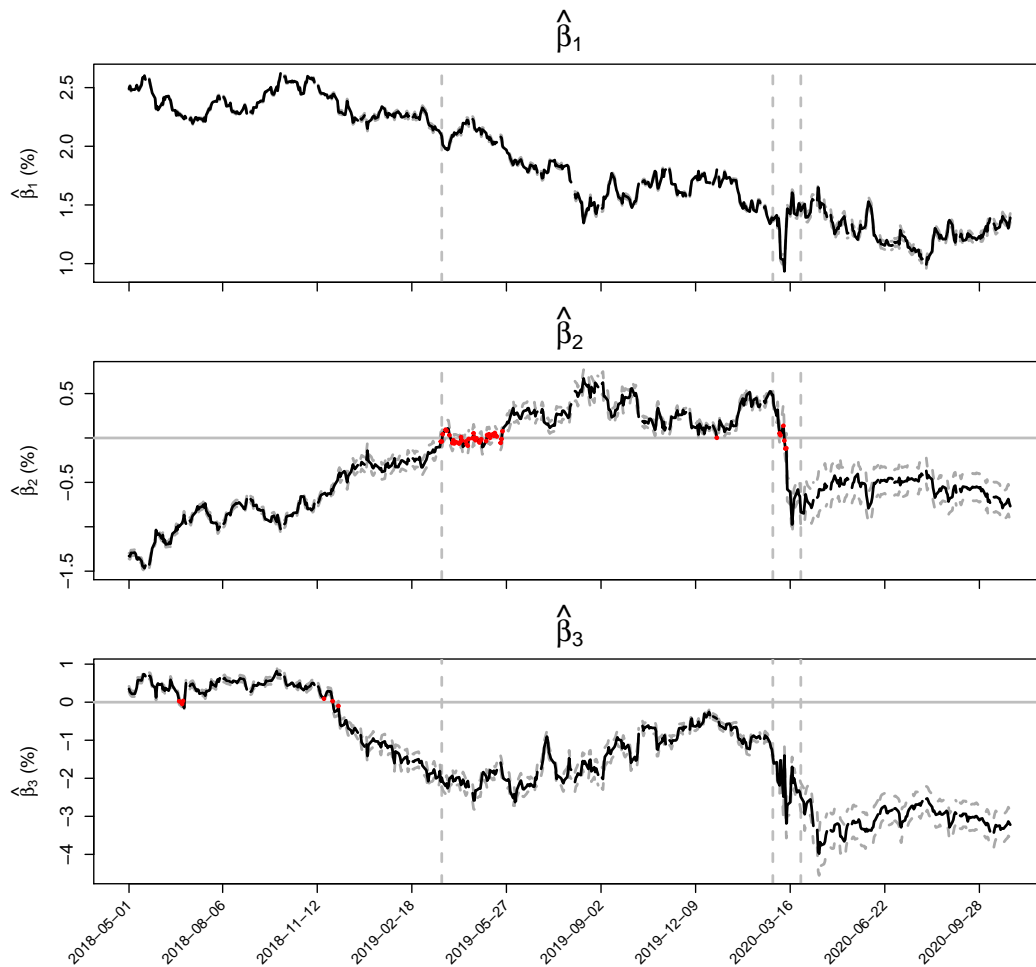

Figure 7: **Nelson and Siegel (1987) factors.** Time series of estimated $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ and their 95% confidence intervals in the Nelson and Siegel (1987) model (with $\lambda_t = 0.0609$). The vertical gray dashed lines show our subdivision in periods. All values are in percentage. Red points correspond to non-significant coefficients (p-value of t-test for $\beta_{j,t} = 0$ larger than 0.05, for $j = 1, 2, 3$). See also Figure A5 in Appendix A2.

can be interpreted as *level*, *slope* and *curvature* factors, respectively. Specifically, $\beta_{1,t}$ coincides with the long-term yield, $\beta_{2,t}$ is closely related with the term spread and $\beta_{3,t}$ is associated with twice the two-year yield minus the sum of the ten-year and three-month yields.

In eq. (1) the decay parameter λ_t is chosen as to maximize the medium-term regressor when $x = 30$ months. This approach leads to $\lambda_t = 0.0609$, as shown in Diebold and Li (2006). The betas are then estimated via ordinary least squares for each day t. Differently from Papailias (2022), we do not allow for time-varying λ_t, because this would make the beta comparison unreliable. Instead, we prefer to stick to the original approach of Diebold and Li (2006). Figure 7 shows the obtained time series of daily beta estimates. Descriptive statistics for the betas in each of the three periods are collected in Table 2. See also Figure A5 in Appendix A2 for the corresponding t-test p-values.

Table 2: **Descriptive statistics of Nelson and Siegel (1987) factors.** Mean, volatility (standard deviation) and relative volatility (coefficient of variation) of $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ in each of the three periods.

	Mean (%)	Volatility (%)	Relative volatility
Period 1			
$\hat{\beta}_{1,t}$	2.3542	0.1224	0.0520
$\hat{\beta}_{2,t}$	-0.7065	0.3532	0.4999
$\hat{\beta}_{3,t}$	-0.1449	0.8546	5.8982
Period 2			
$\hat{\beta}_{1,t}$	1.7471	0.2151	0.1231
$\hat{\beta}_{2,t}$	0.2435	0.1831	0.7521
$\hat{\beta}_{3,t}$	-1.4169	0.6153	0.4342
Period 3			
$\hat{\beta}_{1,t}$	1.2750	0.1339	0.1050
$\hat{\beta}_{2,t}$	-0.5466	0.0992	0.1814
$\hat{\beta}_{3,t}$	-3.0557	0.3131	0.1025

5.2 Results interpretation

The $\beta_{1,t}$ estimates decrease, on average, from the first to the second period, as well as from the second to the third one (Figure 7 and Table 2). Such decrease reflects the declines of mean

long-term yields along the three periods we observed in the top panel of Figure 3. The Bank of Canada cuts of the target rate in March 2020 triggered the decrease of $\beta_{1,t}$ from the second to the third period. In addition, the relative volatility of $\hat{\beta}_{1,t}$ is remarkably high in the second and third periods. This behavior mirrors the one of the relative volatility at far maturities (Figure 3) and reflects the long-run dependence of the level detected by the FPCA. Interestingly, this behavior is not captured by the volatility itself.

The signs of the $\hat{\beta}_{2,t}$ reflect the yield curve inversion of the second period. Indeed, $\hat{\beta}_{2,t}$ is almost always positive in the second period, while it is negative in the other periods (Figure 7 and Table 2). The COVID-19 period features an increasing yield curve, similarly to the first period. The central bank interventions contributed to correct the curve inversion of the second period. In addition, the relative volatility of $\hat{\beta}_{2,t}$ is higher in the second period, mirroring the higher variance explained by the second functional principal component (slope) in the second period, with respect to the other periods.

The behavior of $\hat{\beta}_{3,t}$ permits to better understand the curvature in the three periods. As shown in Diebold and Li (2006), this factor is closely related with twice the two-year yield minus the sum of the ten-year and three-month yields. $\hat{\beta}_{3,t}$ decrease over the three period, on average, capturing a more and more valuable curvature effect. Indeed, in the second period, the curvature is due to the inverted hump in the mean yield curve. In the third period, although the yield curve is increasing, the mean $\hat{\beta}_{3,t}$ is low because of relatively high long-term rates with respect to the two-year yield.

6 Conclusions and further discussion

Our methodology relies on the application of FPCA to the term structure of yields to maturity and it is complemented by the (exponential) regression model of Nelson and Siegel (1987). A simple but key step of the analysis is the split of the sample period, which permits to detect and quantify the different behaviors of the yield curves. Future works involve the development of a time-dependent FPCA, which would permit to follow the evolution in time of the components in a continuous fashion, overcoming the issue of splitting the sample in separate periods. For comparison, the outcomes of (classic) PCA are displayed in Figures A2-A4 in Appendix A1. Results are qualitatively similar to FPCA, but they are less smooth, hence harder to interpret. These differences between FPCA and PCA would be more striking with more noisy and/or more sparsely sampled yield curves, and highlight the advantages of working in a functional framework.

Regarding the Nelson and Siegel (1987) approach, the presence of unit roots in the sequences of $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ can be inspected, as well as the shape of their autocorrelation functions. Since such an analysis is not particularly insightful for understanding the changes in the yield curve during the pandemic, we address these issues in Appendix A2.

In summary, we can draw the following conclusions on the consequences of the Bank of

Canada interventions (due to the first COVID-19 wave) on the term structure of government bond yields.

- During the COVID-19 pandemic, the average term structure of rates is increasing, as it was before March 21, 2019: Bank of Canada interventions contribute to cancel the curve inversion observable between March 22, 2019 and March 3, 2020. In addition, yields at all maturities in the COVID-19 period are lower than in previous periods. The target rate cut by the central bank induced a decrease of rates also at long maturities.

- During the COVID-19 crisis, relative volatility is extremely high for bond yields with close maturities. This reflects a high level of uncertainty about the near future. Moreover, the long-run relative variance is not different from the one of the twelve months before. Despite Bank of Canada stimuli, the same amount of long-run risk is present.

- The level (in FPCA) has the same long-run dependence during the pandemic and in the twelve months before. This provides evidence of the maintained persistence of yields to maturity despite their overall decrease.

- During the pandemic, the slope (in FPCA) keeps the same long-run dependence of the twelve months before. However, it explain less variance and a modest shock in the slope is not able to modify the monotony of the yield curve. Indeed, the increasing monotony of the yield curve induced by the central bank interventions is rather insensitive to shocks in the slope component.

- In the COVID-19 period, a curvature effect, from Nelson and Siegel (1987) model, is present even though the yield curve does not display any hump. This is caused by relatively high long-term yields with respect to medium-term yields.

In a nutshell, the FPCA of the first pandemic wave is similar to the one of the twelve months before, while the mean yield curve is likely to be a downward shift of the mean curve dating back to the period before March 21, 2019. By modifying the target rate and introducing massive stimuli in the economy during COVID-19 outbreak, the Bank of Canada was effective in lowering all yields to maturity and correcting the inverted hump in the curve, but it was not able to affect the (remarkable) long-run risks.

Appendix

A1   Complements on FPCA

Figure A1 shows the functional boxplots of the yield curves on the different time periods described in Subsection 3.1.

Figures A2-A4 contain the outcomes of the classic PCA (for comparison, FPCA results can be found in Figures 4-6 in the main text).
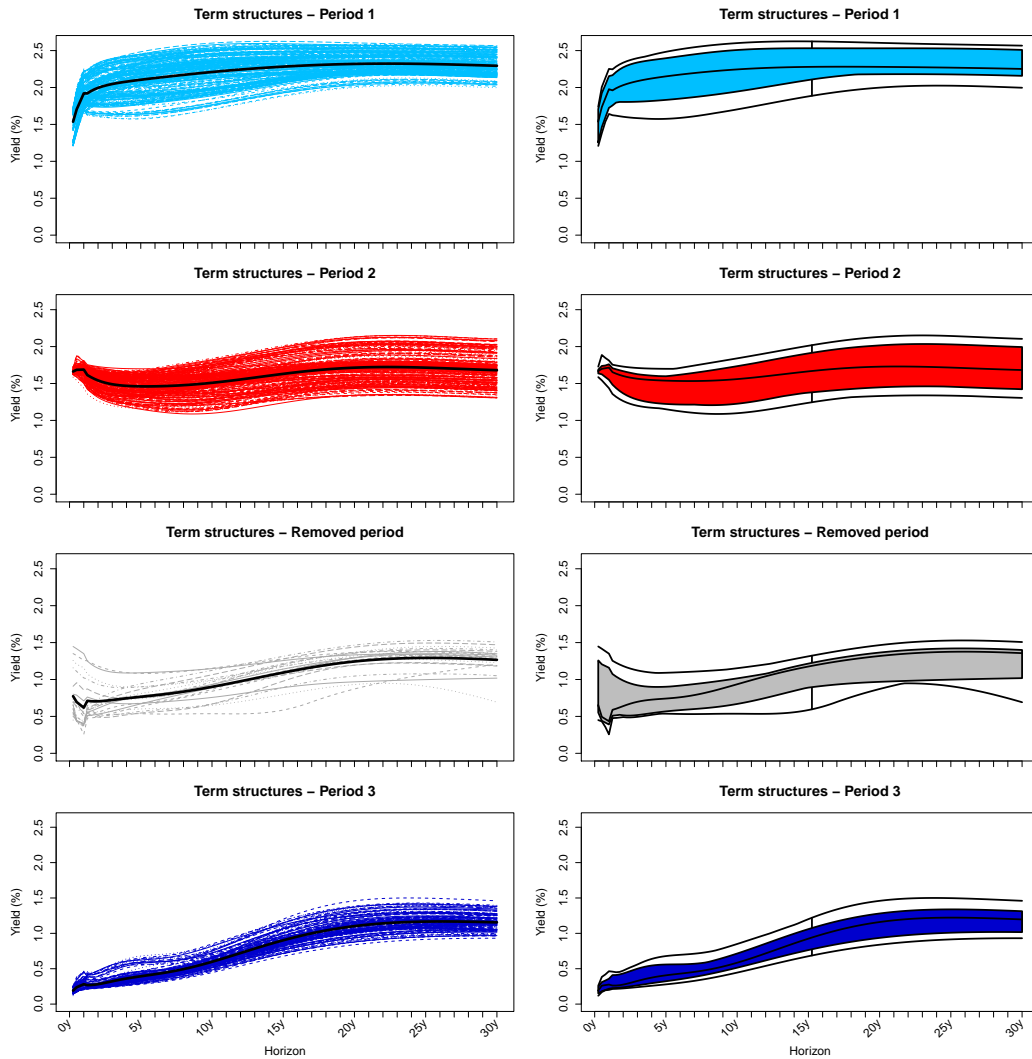

Figure A1: **Functional boxplots of the yield curves.** Daily yield curves (left panels) and corresponding functional boxplots (right panels) in the three periods considered in the analysis, as well as in the removed transition period (February 28 to March 26, 2020). The black lines in the left panels represent the mean yield curve in each period.
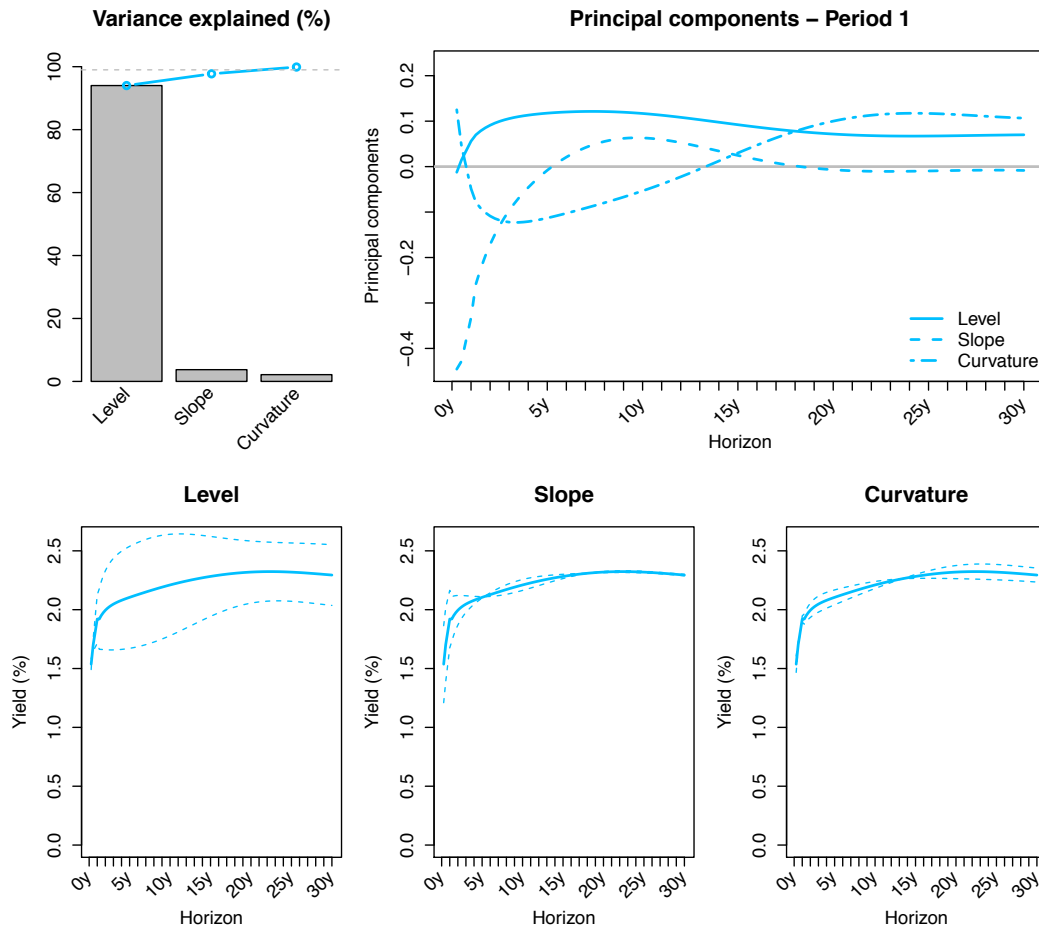
Figure A2: **(Classic) PCA results for the first period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenvectors of such components. The bottom panels represent the raw mean term structures of yields in the first period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

A2 Complements on Nelson and Siegel (1987) model

Figure A5 contains the p-values of the t-test for $\beta_{j,t} = 0$ vs $\beta_{j,t} \neq 0$ in Nelson and Siegel (1987) model, for $j = 1, 2, 3$. Such coefficients are almost always significant (p-value < 0.05). The graphs complement the results of Figure 7 and Table 2.

Table A1 contains the Augmented Dickey–Fuller (ADF) unit root test statistics, as well as the related p-values, for the three factors in each period. The unit root null hypothesis is rejected (in favour of trend-stationarity) at 5% level only for $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ in the third period.
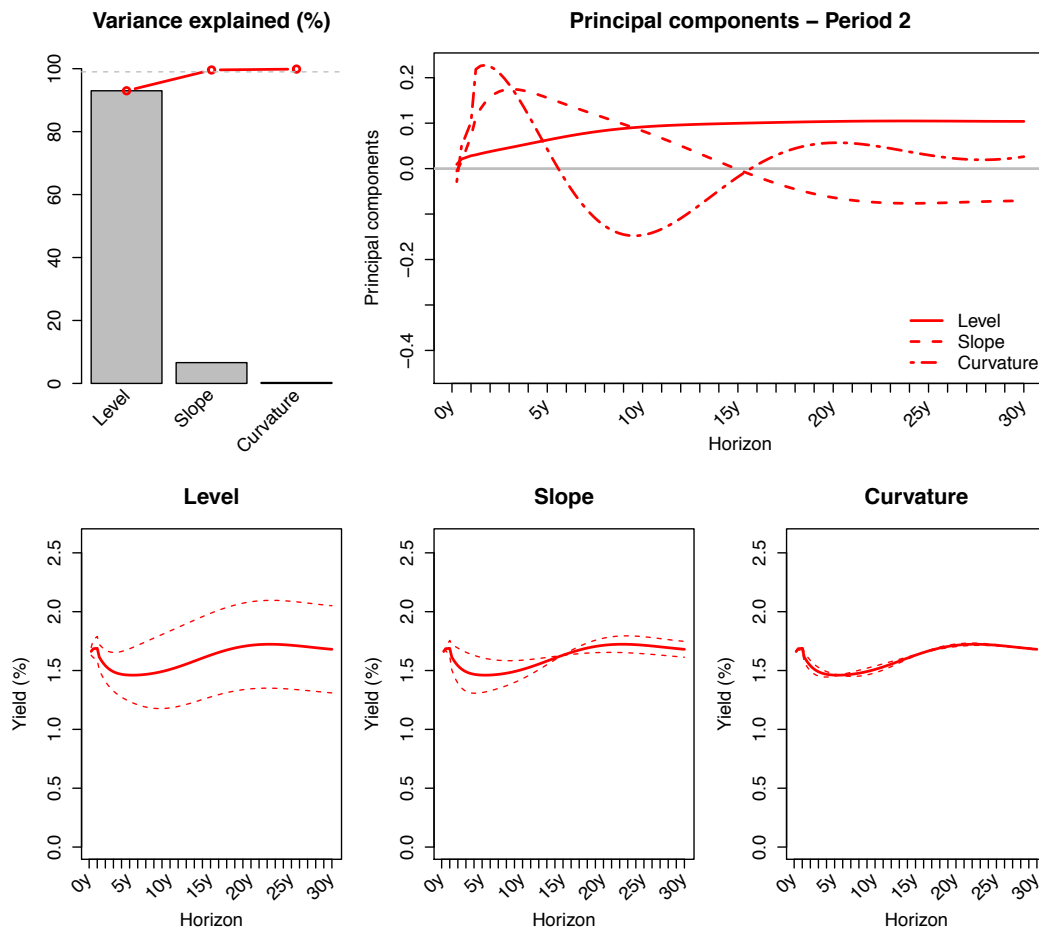
Figure A3: **(Classic) PCA results for the second period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenvectors of such components. The bottom panels represent the raw mean term structures of yields in the second period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

Figures A6-A8 show the correlograms of the factors in the three periods under scrutiny. In the third period, the decay of the autocorrelation of $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ is consistent with the rejection of the unit root hypothesis in the previous ADF test.
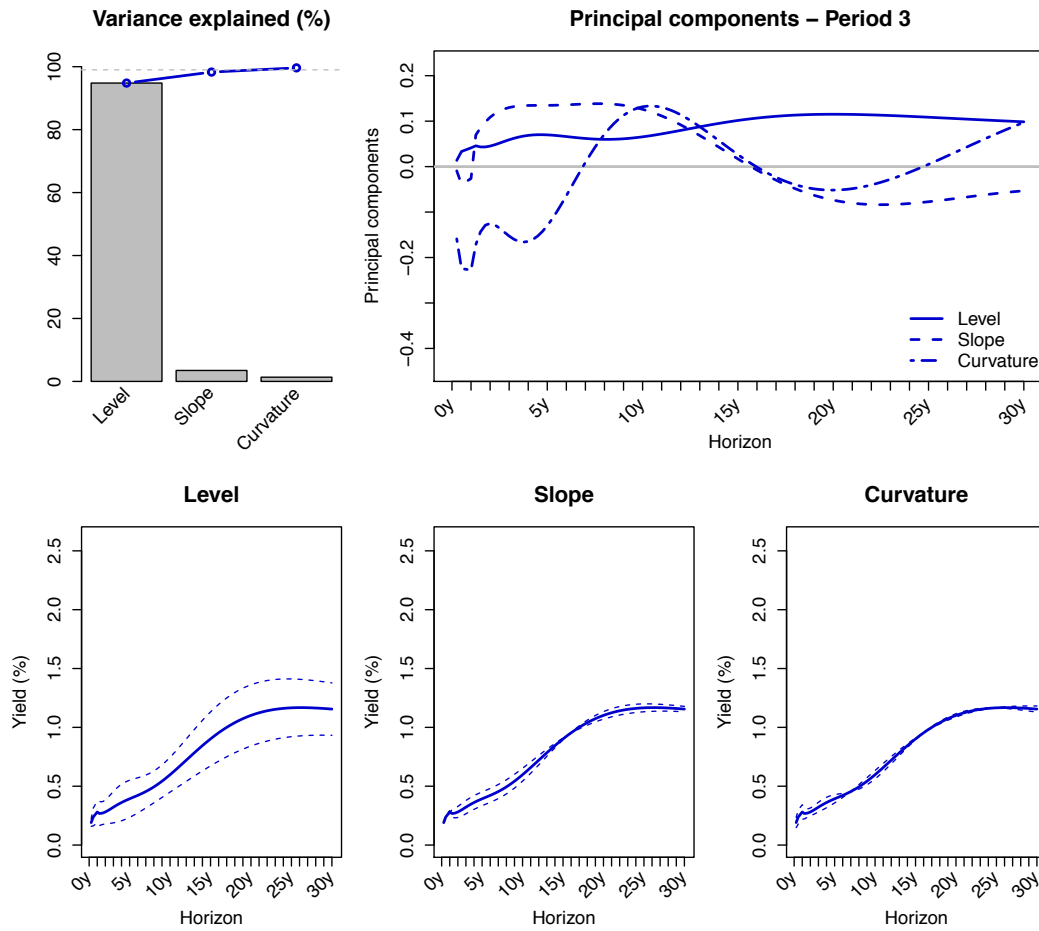
Figure A4: **(Classic) PCA results for the third period.** The top-left barplot shows the variance explained by the first three components (interpreted as level, slope and curvature). The top-right panel displays the eigenvectors of such components. The bottom panels represent the raw mean term structures of yields in the third period, together with the negatively or positively shocked curves obtained by subtracting or adding twice the standard deviation of the component times the component curve. All values are in percentage.

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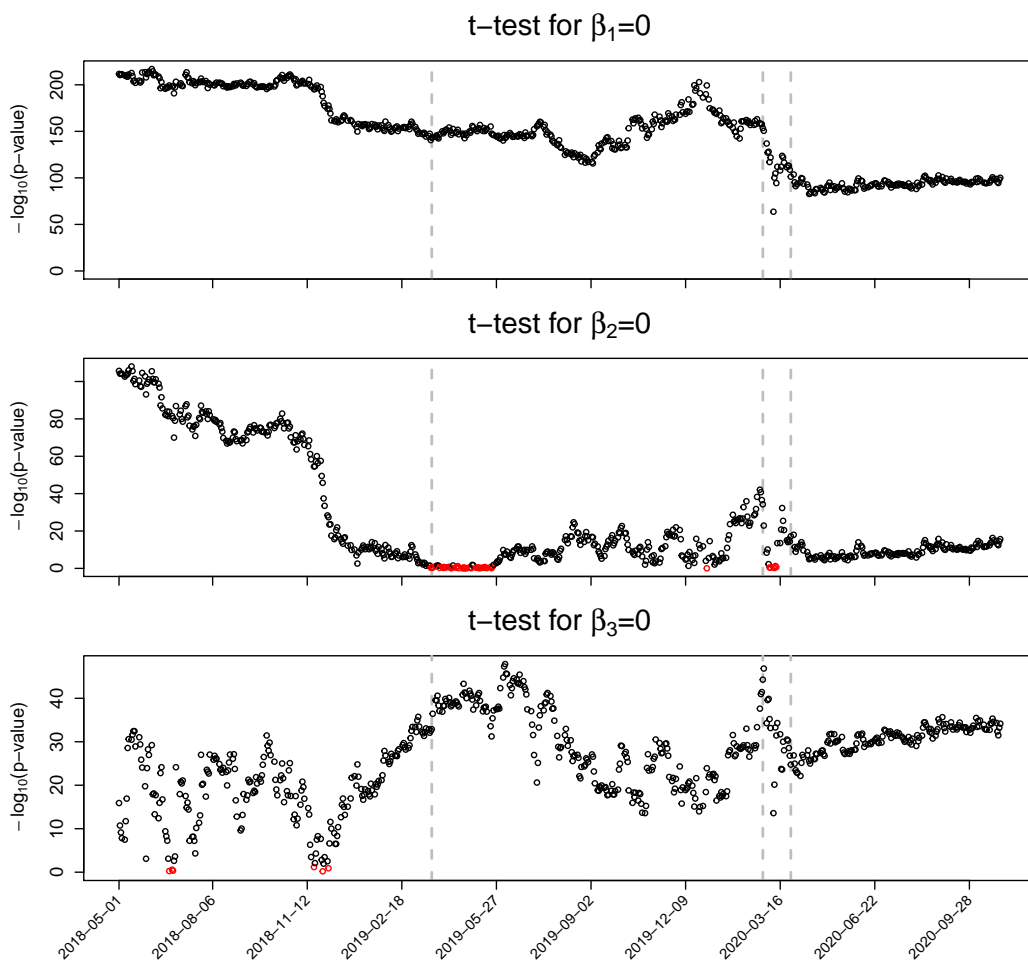

Figure A5: **P-values in Nelson and Siegel (1987) model.** P-values of t-tests for $\beta_{j,t} = 0$, $j = 1, 2, 3$ in Nelson and Siegel (1987) model. Red points correspond to non-significant coefficients (p-value > 0.05). The vertical gray dashed lines show our subdivision in periods.

Table A1: **ADF test for Nelson and Siegel (1987) factors.** ADF test statistics and p-values for $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ in each of the three periods. The maximum lag used in each test is chosen as to minimize the BIC. P-values are computed using the `ADF.test` function of the `tseries` R package. [(*)] indicates significance at 5% level.

	ADF test statistics	ADF test p-value
Period 1		
$\hat{\beta}_{1,t}$	-1.8153	0.6535
$\hat{\beta}_{2,t}$	-2.5118	0.3611
$\hat{\beta}_{3,t}$	-1.7231	0.6922
Period 2		
$\hat{\beta}_{1,t}$	-2.3295	0.4374
$\hat{\beta}_{2,t}$	-2.3413	0.4325
$\hat{\beta}_{3,t}$	-3.0704	0.1260
Period 3		
$\hat{\beta}_{1,t}$	-2.9688	0.1744
$\hat{\beta}_{2,t}$	-3.5971	0.0366[(*)]
$\hat{\beta}_{3,t}$	-3.5531	0.0405[(*)]

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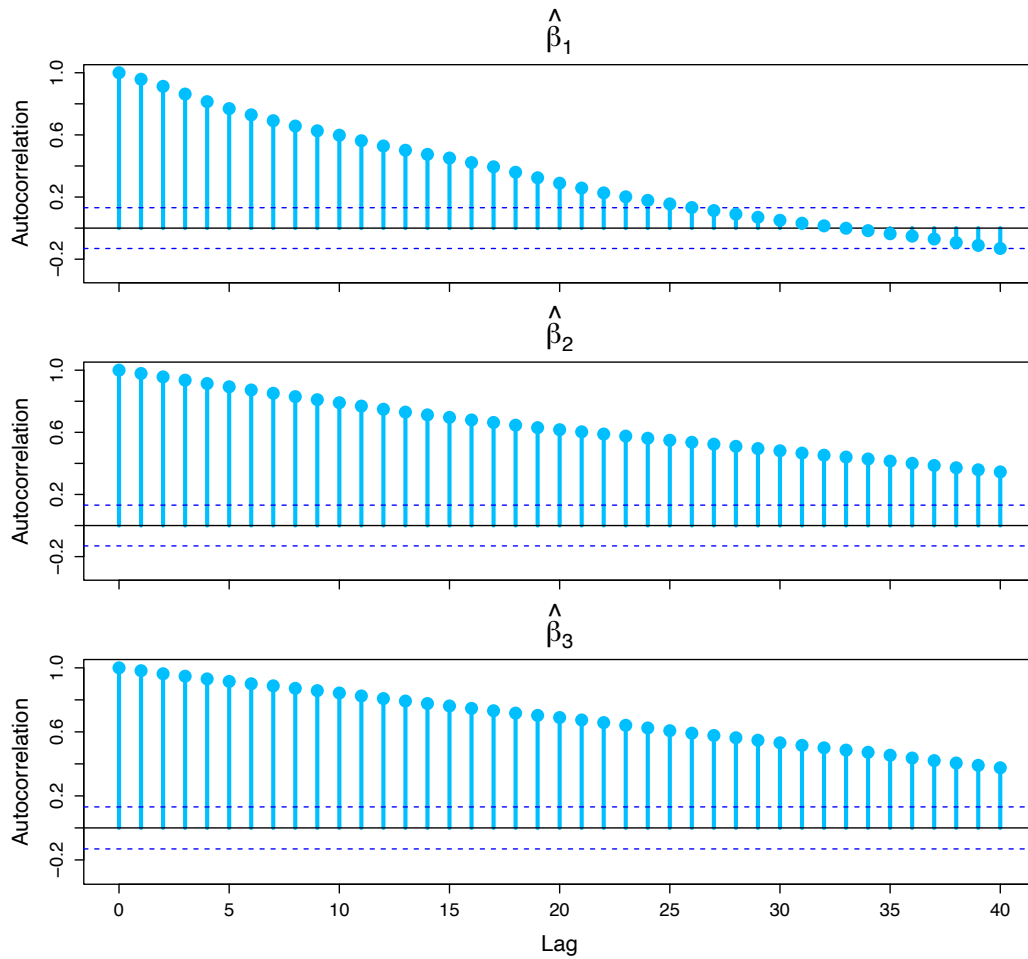
Figure A6: **Autocorrelation of Nelson and Siegel (1987) factors in the first period.** Sample autocorrelation functions for $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ in the first period with approximate 95% confidence intervals based on an uncorrelated series.

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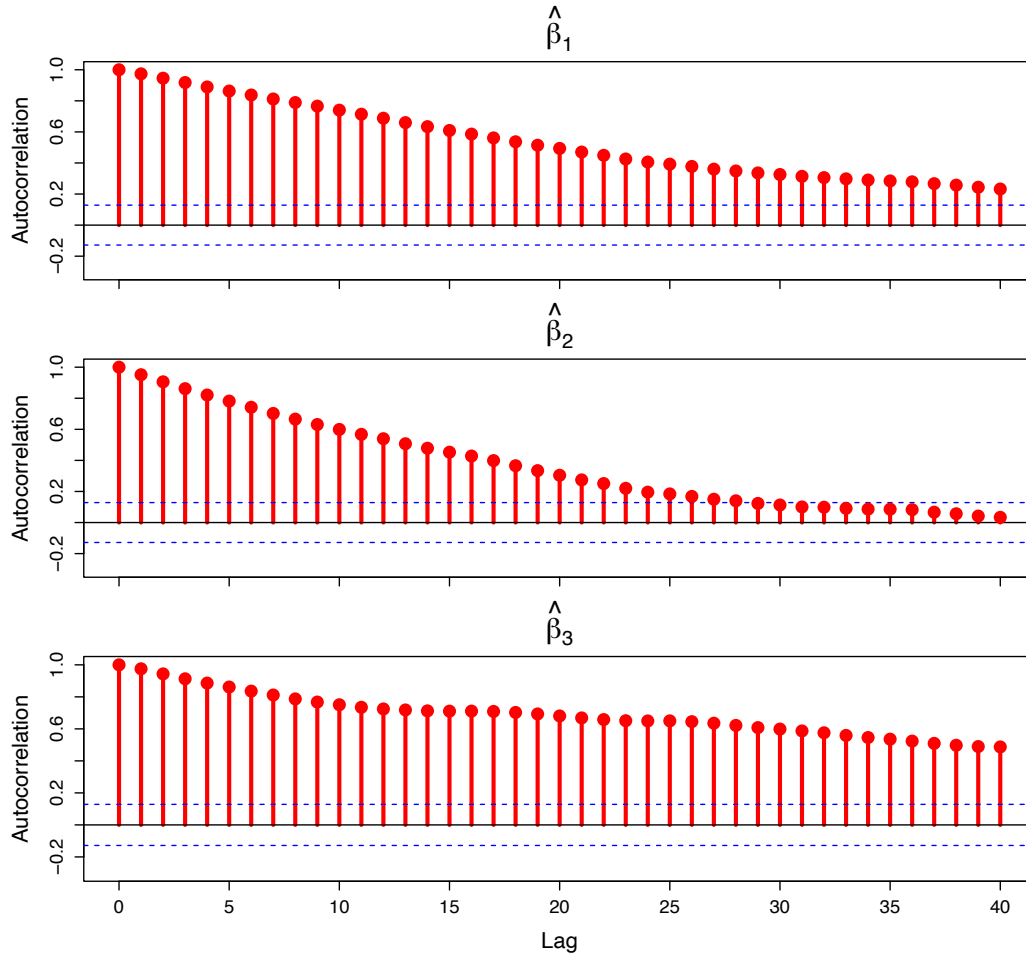
$$\hat{\beta}_1$$

$$\hat{\beta}_2$$

$$\hat{\beta}_3$$

Figure A7: **Autocorrelation of Nelson and Siegel (1987) factors in the second period.** Sample autocorrelation functions for $\hat{\beta}_{1,t}, \hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ in the second period with approximate 95% confidence intervals based on an uncorrelated series.

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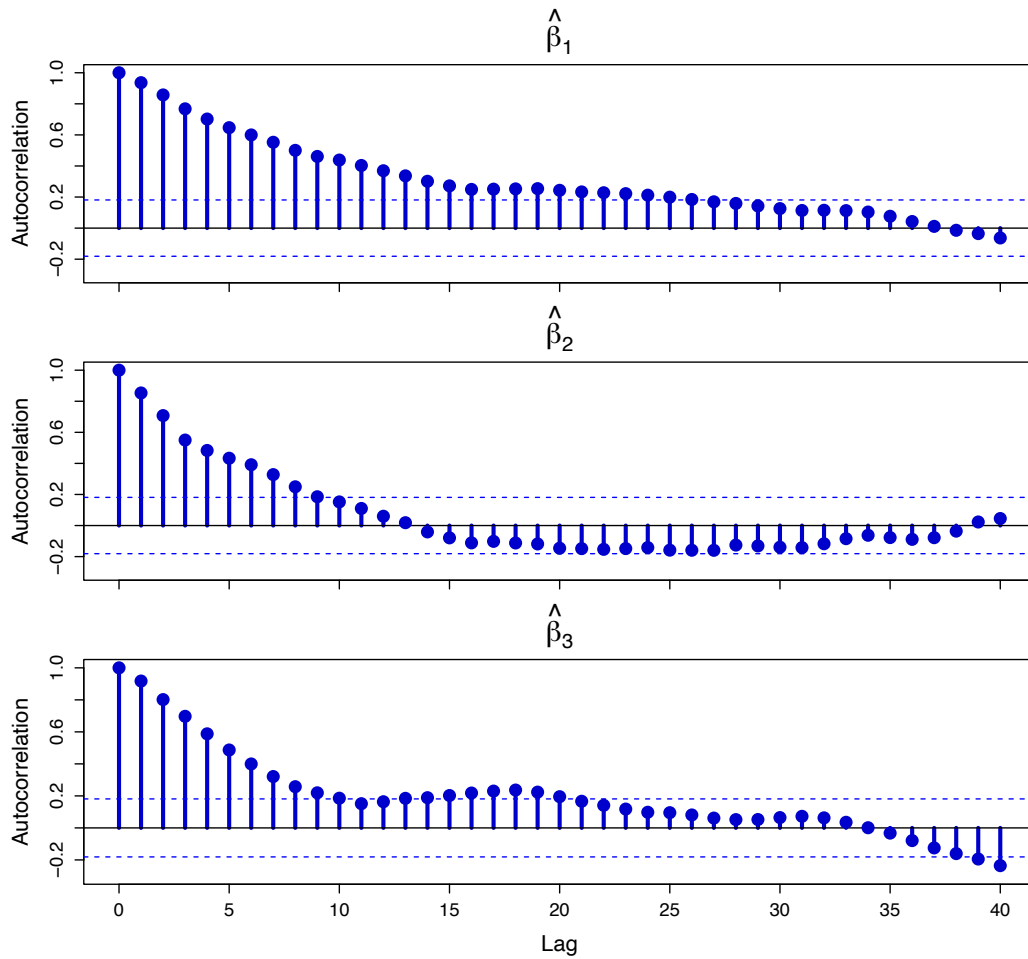
Figure A8: **Autocorrelation of Nelson and Siegel (1987) factors in the third period.** Sample autocorrelation functions for $\hat{\beta}_{1,t}$, $\hat{\beta}_{2,t}$ and $\hat{\beta}_{3,t}$ in the third period with approximate 95% confidence intervals based on an uncorrelated series.

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