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Estimating the fundamental value of sports clubs and stadia: Application to Panathinaikos FC and Leoforos

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We propose a fundamental valuation model for sports clubs and stadia using discounted (adjusted) revenues. We argue that a sports club is a "quasi firm" that aims to balance budgets, achieve an efficient allocation of financial resources, and maximize revenues. Under this objective the sports club's welfare and value are maximized. Then we offer a method for estimating the value of a sports club's stadium. The proposed valuation model can be useful during acquisition negotiations or for assessing managerial performance. Combining the proposed model with stochastic Monte Carlo simulations, we estimate the brand-name value and the club's total value of the football team of Panathinaikos, as well as the value of its iconic home ground, Apostolos Nikolaidis Stadium (known as Leoforos) located in the heart of Athens at Alexandra's Avenue

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1 Introduction

The concept of fundamental valuation is very important in finance when it comes to assessing the true value of a business, especially for mergers and acquisitions. However, fundamental valuation remains understudied for the case of sports clubs in the sports finance literature. We fill this gap by proposing a fundamental valuation model for sports clubs and their stadia.

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The debate in sports clubs between profit maximization and utility maximization began with Rottenberg (1956) and Sloane (1971); for more details see Fort and Quirk (2004), Sandy et al. (2004), Kesenne and Pauwels (2006), Kesenne (2007), Garcia-del-Barrio and Szymanski (2009), Dietl et al. (2009), Fort (2015). Many years later, Madden (2012), and Madden and Robinson (2012) introduced fan welfare maximization given increased involvement of supporters' trusts, and this is an addition to win maximization (equivalent to maximizing team quality) within utility maximization. Finally, Terrien et al. (2017) support that football clubs may move across three different objectives: profit maximization under sporting constraint (I), win maximization under hard budget constraint (II), and win maximization under soft budget constraint (III).

As stated earlier, fundamental valuation of firms is one of the most crucial themes in the finance literature, e.g. for mergers and acquisitions (M&As) and investors' buy/sell strategies. In fact, there is a plethora of valuation models (see Damodaran, 2012, and Fernandez, 2019, for an extensive review), which are applied to both financial and non-financial firms either with a rich history or with a short life and for different stages of the economic cycle (market buoyancy and financial stress). When it comes to the special case of sports industry (e.g. Neale, 1964; Downward and Dawson, 2000; Howard and Humphreys, 2008; Downward et al. 2009; Fort, 2010; Kesenne 2015; Howard and Crompton 2018; Leeds et al. 2018), *Brand Finance*, *Forbes*, and *KPMG* have attempted to estimate the value of brand name, the team value, and the enterprise value, respectively, of a football team employing the team's financial performance. *Forbes* also provides estimates for the brand value of sports clubs. *Statista* briefly describes the methodologies for these valuations and provides estimates on annual basis see here, here and here); however, details and assumptions for the estimation procedure are not available to the public.[1]

Regarding *Forbes* values, these are calculated as revenues' multiples (see Scelles et al., 2016, Geckil et al., 2007, Fort, 2006, Vine, 2004) taking into account club-specific characteristics (such as past transactions, expenses, debt, and stadium deals), and the employed methodology is proprietary. *Football Benchmark* (now an independent entity after the spinoff of *KPMG*'s Sports Centre of Excellence) estimates the enterprise value of sports clubs, based on revenues' multiples too, using a proprietary 5-pillar valuation algorithm that also considers club-specific characteristics. Finally, when it comes to *Brand Finance*, the royalty relief methodology is used for the estimation of the brand-name value, but details on models' parameterization are unavailable to the public too.

There are several papers deciphering the "black box" of *Forbes* methodology and finding determinants of *Forbes* estimated franchise values (see among others Alexander and Kern,

[1] One can see for 2024 the most valuable and strongest football club brands by *Brand Finance* (here) and the world's 10 most valuable soccer teams by *Forbes* (here).

2004; Vine 2004; Miller, 2007, 2009; Büschemann and Deutscher 2011, Scelles et al., 2013, 2016; DeSantis, 2018). On the other hand, some authors examine the actual sale prices and find drivers of transaction prices. For example, Humphreys and Mondello (2008), and Humphreys and Lee (2010) use prices paid for sports franchises in their analyses. Fort (2006) employs both actual franchise sale prices and estimated franchise values and performs an unconditional analysis for the Major League Baseball (MLB).

Most of these papers deal with North American professional sports, where the sports clubs are generally profit maximizers; Ziets and Habber (2008) address the question of how much the professional sports teams in North America are worth and discuss difficulties in valuing such sports clubs that are privately held and not listed in the stock market. In contrast, Scelles et al. (2013, 2016) focus on European football clubs that are usually loss-generating entities.[2]

Buraimo and Simmons (2008) examine the profitability of sports teams. They argue that the European professional football clubs regularly encounter financial problems and operate in a very different environment than North American professional sports teams. Furthermore, Tiscini and Dello Strologo (2016) provide evidence that the average financial results are negative for the listed European football clubs and that their value is related to revenues rather than income. Terrien et al. (2017) show that at least one third of the French football clubs record operating losses. Markham (2013) argues that English football clubs are not profitable, while Storm and Nielsen (2012) argue that many football clubs in European top leagues face persistent deficits, rising debt, and abnormally high survival rates. Finally, Madden (2012), and Madden and Robinson (2012) argue that wealthy club owners in European football are willing to forego profits to produce champion teams.

Next, there are two valuation models for sports clubs by Markham (2013) and Damodaran (2014) based on profit maximization. Markham (2013) proposes a multivariate model to value the English Premier League football clubs based on revenues, book value of equity, net income, players' wages, and stadium capacity utilization. Besides profit maximization, this model

[2] According to UEFA (2012) in the 2010/2011 season, 63%, 55% and 38% of European top-divisions clubs, respectively, reported operating losses (compared with 51% in 2007), losses, and negative book equity values. Similarly, according to a more recent report (UEFA, 2023), 55% and 25% of early-reporting clubs recorded pre-tax losses and negative book equity values, respectively, in the 2021/2022 season. Additionally, from this report, 68% of all reporting clubs suffered pre-tax losses in the 2019/2020 and 2020/2021 seasons (up from 45% in the 2018/2019 season) and 38% of all reporting clubs recorded negative book equity values. The period 2009-2019 is characterized by declining aggregate pre-tax losses (with the exemption of the profitable seasons 2016/2017 and 2017/2018), but from 2020 to 2022 elevated losses were recorded. Therefore, it seems that the implementation of financial fair play regulations (now financial sustainability regulations) at the start of 2011/2012 season has had a little effect at aggregate level, in terms of reducing the percentage of teams facing losses and negative book equity values. Ahtiainen and Jarva (2022), in the same vein, argue that the implementation of the "financial fair play break even rule" has been at best modest, and they cannot rule out that any observed improved performance is due to the recovery of the 07-09 financial crisis.

significantly encourages the creation of a strong capital base to increase the club's value. Damodaran (2014) uses a one-stage fundamental valuation model with discounted free cash flows to the firm (FCFF) to estimate the franchise value of Los Angeles Clippers.

These two models are shareholder-oriented, aiming to maximize shareholders' financial interests, and treat sports clubs as profit maximizers like standard firms. However, making a profit involves the creation of surpluses whose greatest part is sooner or later destined to increase the wealth of sports clubs' shareholders. Instead, such surpluses could further strengthen the team (e.g. motivating players with higher salaries, or hiring top-quality players, or investing more in academy players and facilities) and increase its match performance. Therefore, reducing or depleting any surplus –given wise spending– reduces "waste" in valuation terms and enhances the club's welfare.

Next, there is a strand of literature focusing on estimation and prediction of transfer fees of football players, with either linear models or machine learning techniques, given a rich set of controls including club-specific and player-specific factors (see, for example, McHale and Holmes, 2023, Poli et al., 2022). Consequently, a team's transfer value could be computed as the sum of individual players' estimated transfer values.[3]

Achieving excellent match performance while avoiding financial problems is key to maximizing a sports club's welfare. In this paper, we apply fundamental analysis and financial valuation tools to estimate sports clubs' value. To this end, we propose a framework for sports clubs in which balanced budgets are preferred and profit maximization is abandoned.

To our best knowledge at the time of writing this paper, there is a gap in the literature on sports clubs' fundamental valuation as there are no fundamental valuation models whose primary goal is to maximize the sports club's welfare and value. In this paper, we aim to fill this gap by proposing such a model and analyzing the valuation of sports clubs and their stadia.

According to our proposed framework, balanced budgets are a necessary condition for maximizing the club's welfare and value. The other necessary condition is the efficient allocation of the club's financial resources ("wise spending"); a condition which also implies maximization of the club's revenues, as we show later. Therefore, (1) if there are profits, or (2) if there are no equity injections to cover any losses or dividends, or (3) if there is no wise spending, then no maximum value and welfare can be attained. To sum up, maximizing value and welfare is a sufficient and necessary condition for balanced budgets and wise spending.

The rest of the paper is organized as follows. Section 2 presents our proposed framework of fundamental valuation. Section 3 is dedicated to discussion and managerial implications of the presented fundamental valuation model. Section 4 illustrates our methodology in an empirical application for Panathinaikos FC. Finally, Section 5 concludes.

[3] Regarding the popular crowd-sourced market values from *transfermarkt.com*, Coates and Parshakov (2022) provide evidence that *transfermarkt values* are biased predictors of actual transfer fees.

2 A proposed framework of fundamental valuation

We build a framework for maximizing the sports club's welfare (in terms of conquering trophies) and value, based on three core principles:

2.1 Management under a benevolent manager

The club's chairman or major shareholder hires a benevolent and experienced manager[4] that shows respect to the club's history and fans.[5] Such a manager takes decisions with the club's best interest in mind and applies the triptych of success: "the team, the fans and the management" united for the club's welfare. Alternatively, the manager follows the sports maxim: *salus "club" suprema lex esto*, i.e. the good of the club shall be the supreme law for the manager. Also, the benevolent manager avoids overpricing policies on tickets, pursuits a match performance as excellent as possible for the club subject to the "fair play rules" and "not winning at all costs", exhibits aversion to debt accumulation and negative book equity values, and his incentives are aligned with these of the fans: to win trophies. Next, the club's head must not apply "divide and rule" policies to the fans, or create handpicked job openings for the core and most active part of the fans to control them and earn their discretion for any management irregularities or financial misconducts. Furthermore, the manager and the major shareholder are administrators and not the club's owners, while they must view their role as custodians and keepers of the club. The sports club belongs only to its fans. However, in modern sports, i.e. after the transition from amateurism to professionalism, a club without a robust financial outlook has unfortunately no (bright) future in celebrating titles.

2.2 No dividends paid, or share repurchase programs approved

The shareholders of sports clubs invest voluntarily in the club aiming to increase the budget and better match seasons.[6] At the same time, they should expect no reimbursement through dividend policy and the invested capital should be treated as a sunk cost for their portfolio. However, the shareholders, or the major shareholder, may receive an implicit, or indirect,

[4] Shareholders with the required "know-how" and experience could also run the club.

[5] The term "benevolent" is also found in macroeconomics, and specifically in the field of DSGE models where the goal of a benevolent social planner is to maximize households' utilities.

[6] Madden (2015) and Lang et al. (2011) address ownership of benefactor ("sugar daddies") who invest great sums of money in sports clubs. This is in accordance with Damodaran (2014) who argues that sports clubs could be seen as luxury assets, and "ego and pride premia" may be embedded in sports franchise acquisitions due to managerial hubris. In fact, Vine (2004) detects a 27% premium of transaction prices relative to *Forbes* estimated values for North American professional sports and suggests that it may stem from "ego factor" executives. Also, ego-driven desires of investors who want to own "trophy assets" are discussed in Football Benchmark (2022).

benefit to their main business from monetizing the club's reputation, brand name, and fans[7]. Specifically, there can be a positive effect (specific synergies) on the shareholders' owned firms because the club's fans may prefer the products/services of these firms to those of any rival firms, leading to greater exposure and elevated profitability for the shareholders' other businesses (Football Benchmark, 2022). Such behavior could be justified under the framework of investor sentiment and behavioral finance.

The shareholders, or major shareholder, may also experience synergies or receive an implicit benefit[8] that is extremely difficult to evaluate, from leverage in negotiations *vis-a-vis* the government and political lobbying[9] on the grounds that the fans of the most prominent and popular sports clubs are "socially big" (see Storm and Nielsen, 2012) and constitute a considerable portion of the electoral body[10]. Such a hidden benefit could justify vast amounts spent on equity injections to cover losses and mismanagement, even if covering losses of a sports club constitutes a bleeding practice for the shareholders' portfolio.

Also, the fans must never be financially dependent on the head of the club and should never require shareholders to increase the club's budget through capital increases. On the other hand, if there are continuously poor match seasons, or evidence of mismanagement, the fans must protest and request a change in the head of the club. However, this could happen if the fans are always independent of the club's shareholders, management, and chairman.

2.3 No regulatory obligation to build up capital buffers like a bank

Storm and Nielsen (2012) argue that football clubs are "too big to fail" from a social perspective and not from a financial point of view. A sports club default is not a systemic event and does not affect the financial markets as if there was a traditional bank run, or a financial meltdown like the 2007-2009 global crisis. Capital buffers help a bank remain solvent in case of severe losses and continue its financial intermediation role. On the other hand, a sports club could be run with small or large book equity values.

Next, three claims arise from the core principles above, and reasoning is provided for each of these claims.

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[7] Wakefield (2016) argues that fan passion may predict traditional media consumption and social media behavior.

[8] Garcia-del-Barrio and Szymanski (2009) argue that the shareholders may be willing to tolerate losses because these losses could be offset by profits in other businesses.

[9] See, for example, Acemoglu et al. (2016), Bertsatos et al. (2023), Borisov et al. (2016), and Faccio (2006) for the effects of lobbying practices and corruption on firm value.

[10] Coates and Humphreys (2008) conclude that sports subsidization is undesirable and point out the connection between the political value for elected officials and the rent-seeking behavior for professional athletes and sports team owners.

Claim #1

The sports club's benevolent management abandons the profit maximization behavior of an ordinary firm and increases the club's welfare, i.e. achieving the best possible match performance without facing financial difficulties, by adopting a different strategic planning: that of balanced budgets.

Reasoning:

Given that there is no dividend policy, either conventional with the standard dividends or unconventional with stock buybacks, and that there is no regulatory obligation for increased capital base, generating profits is not the best option for a sports club. Besides, profit maximization does not imply maximization of the sports club's welfare. Maximizing profits involves the creation of lucrative surpluses that could be distributed to shareholders, or strengthening the capital base, as in ordinary firms. But such surpluses will potentially reduce the total amount spent on strengthening the team. For example, surpluses could be depleted by setting higher salaries for the players, or introducing bonus schemes for the players upon the achievement of specific targets, or hiring players of higher quality, or investing more in academy players and facilities. Such or similar actions could improve the sports club's match performance both in the short run and the long run. Consequently, reducing surpluses by increasing costs –given wise spending– reduces "waste" in valuation terms and increases the sports club's welfare.

Next, like a standard firm, a sports club cannot sustain periods with financial losses since they contribute to debt accumulation and equity crunches, implying a greater probability of default. For example, there is no point in hiring athletes whose contracts cannot be financially met by the club, leading to impositions of sanctions that tarnish the club's brand name and reputation worldwide. Therefore, reducing losses by increasing revenues streams, or by adjusting costs to the point of adequate funding, reduces "waste" in valuation terms and increases the club's welfare.

Consequently, a sports club's benevolent manager aims to increase and maximize the club's welfare with balanced budgets.[11, 12] Moreover, balanced budgets could be still achieved *ex*

[11] A balanced budget is also satisfied when revenues (including capital increases by the shareholders) and costs (including corporate taxes and taxation in contracts) are zero. However, this is a degenerate case with no interest.

[12] Garcia-del-Barrio and Szymanski (2009) show that football clubs in the English and Spanish leagues adopt win maximization subject to zero-profit constraint rather than profit maximization. In the same vein, Kesenne (2007), employs win maximization subject to zero-profit budget constraint for the analysis of European football. Madden (2012) compares fan welfare maximization and win maximization with non-negative profits, with profit maximization, whilst Madden and Robinson (2012) employ a weighted average function of profit, win percentage and fan welfare subject to non-negative profits.

post, as we will show later, if any financial losses (or even dividend payments) are covered by share capital increases and any profits are depleted over a fixed time period.

Claim #2

Efficient allocation of revenues implies first revenue maximization and second, maximization of the sports club's welfare and value.

Reasoning:

Suppose that the manager is efficient and optimally utilizes the money-generating ability of the club, i.e. spends these cash inflows as efficiently as possible. For example, this excludes the probability of any unjustified overpricing or "rent extraction" during contracts signing with the athletes and their managers or, presupposes a scouting team with specialists covering many countries worldwide to find new athletes or, presupposes qualified personnel to develop young players internally. If so, the club's financial resources (fans, broadcasting, sponsorships, and donations) are optimally utilized and in the next season the financial supporters will be willing to contribute more (increased attendance given a benevolent manager and no overpriced tickets[13], greater passion and fan engagement[14], greater number of social media fans, more commercials and sponsorships, greater broadcasting revenues) to the benevolent manager, who does truly care about the club's good and pursues the club's welfare. A virtuous cycle begins between match performance and various financial tools.[15] Namely, more favorable deals with sponsors, and successful membership campaigns. Consequently, optimality in the transformation process of cash inflows to cash outflows implies revenue maximization. Alternatively, the manager could maximize costs given wise spending and sufficient financing, i.e. increase the costs up to the point you finance them whilst avoiding unjustified overpricing. Consequently, given balanced budgets, wise spending, and revenue maximization by the benevolent club's manager, the maximization of the sports club's welfare is accomplished.

Before we discuss value maximization for a sports club, we need first to propose a valuation model. To this purpose we rely on the framework of discounted cash flows for firm fundamental valuation. The valuation process involves discounting the future expected cash flows, where their sum constitutes the value of the examined firm.

[13] Conceptually, given no overpriced tickets one could claim that our proposed framework involves an intersection of win maximization and fan welfare maximization as the club's objective, subject to balanced budgets. See, for example, the theoretical micro-founded model of Madden and Robinson (2012) with zero weighting of profits in the utility function subject to zero profits.

[14] Wakefield (2016) supports that fan passion is a strong predictor of attendance, and Scelles et al. (2017) expect that social media fans could positively impact the value of sports clubs (through revenues generation possibilities).

[15] Such financial tools could involve, for example, team-specific special bonds or team-specific digital currency.

$$FVE_0 = \lim_{T\to\infty} \sum_{t=1}^{T} E(DCF_t) = \lim_{T\to\infty} \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t} \quad (1)$$

where, $E(DCF_t)$ is the expected discounted cash flow at period t, CF_t is the cash flows at period t, r is the discount rate and FVE_0 is the current fundamental value of equity. However, below we show that the most famous and widely used valuation models (see, for example, Damodaran, 2012, Fernandez, 2019, Sapountzoglou and Pentotis, 2017, for a comprehensive literature review) do not apply for the case of sports clubs.

The valuation of sports clubs could not be done with a Dividend Discount Model (DDM) since it contradicts the basic principle for dividends and sports clubs. Gordon and Shapiro (1956), and Gordon (1959) introduced the DDM with a constant growth rate, also known as "Gordon Model", for valuing a stock and since then many variants of DDM with different growth regimes were adopted.[16] DDM estimates the fundamental value of equity based on dividends as the cash flows to be discounted. A DDM-oriented value of equity is increasing in dividends and since a sports club's intention is a zero-dividend policy, the value is automatically pinned down to zero.

Even if DDM was employed to accurately value (listed) sports clubs that may constantly pay dividends to their shareholders, there could not be a consistent and universal comparison to values of non-dividend-paying sports clubs because such values would inevitably be obtained from another valuation model. Moreover, Markham (2013) argues that traditionally few sports clubs pay dividends to their shareholders.

Next, a sports club's valuation could also not be performed with a Residual Income Model (RIM)[17], which is also known as the excess returns model, since it contradicts "Claim #1" of balanced budgets.

$$FVE_0 = BVE_0 + \lim_{T\to\infty} \sum_{t=1}^{T} \frac{RI_t}{(1+r)^t} \quad (2)$$

where, BVE_0 is the current book value of equity, RI_t denotes the residual income at period t, and r is the cost of equity (as in the case of DDM). A RIM-based value of equity is increasing in profits or the excess returns and given the balanced-budget behavior of a sports club, the fundamental value of equity becomes equal to the book value of equity. As a result, the fundamental value of equity is worth the current book value of equity and may exhibit great volatility (negative equity values are also recorded; see e.g. UEFA, 2023). For example, if the current book equity is €500 thousand and in the next month there is a scheduled capital increase

[16] For example, Bertsatos and Sakellaris (2016) introduce the Dynamic Dividend Discount Model, or 3D model, which is an econometric dynamic model and has been further extended by Bertsatos et al. (2017, 2022).

[17] See Ohlson (1995), Feltham and Ohlson (1995), Frankel and Lee (1998).

by the major shareholder of €5 million then, the new fundamental value according to RIM will be eleven times up! However, what if the club has a great steady annual streamline of revenues, stemming from various sponsorships and increased attendance, of €80 million? The fundamental value of equity (€0.5 or €5.5 million) does not incorporate cash inflows and outflows, and just depends on book equity. According to RIM and our framework, a prominent sports club with a small value of book equity and large revenues faces a smaller value than a newly established sports club with a higher capital base and much smaller revenues! As a result, RIM could not be used in our framework.

The Free Cash Flows to the Firm (FCFF) model involves discounting the future FCFF with the weighted average cost of capital (WACC), and the Free Cash Flows to the Equity (FCFE) model discounts the future FCFE with the cost of equity (COE). The calculation of FCFF and FCFE heavily relies on earnings before interest and taxes, and on net income, respectively. Therefore, they are not compatible with loss-generating clubs as there could be no positive cash flows to discount back for the calculation of the present value. Earnings are the major component of FCFF and FCFE. However, since we do not allow for profits or uncovered losses, earnings and therefore the free cash flows, are inappropriate for our framework, even if the other components of the free cash flows are defined and measured accurately for a sports club. As a result, any estimated value from a FCFF or FCFE model could be spurious and misleading.

Given the difficulties of the aforementioned models, which rely on profit maximization to increase intrinsic value, for the special case of sports clubs, in this paper we propose an alternative fundamental valuation model based on (adjusted) revenues as the cash flow of interest. The value of the club is the sum of the expected discounted revenues. We make a wise guess that the following argument is value-maximizer.

$$V_0 = \lim_{T\to\infty} \sum_{t=1}^{T} \left[\frac{min(R_t, C_t)}{(1+r)^t} + \frac{max(R_t - C_t, 0)}{(1+r)^{t+k}} \right] \quad (3)$$

where, r is the discount rate, R is the revenues, C is the costs and $k = \{1,2,\dots\}$. The aforementioned argument penalizes surpluses and losses on the one hand, and on the other hand it rewards balanced budgets. Regarding surpluses, we assume they are depleted in the next periods either gradually or fully.

A balanced-budget planning implies that revenues equal costs. Given Equation (3), we define the value based on periods of balanced budgeting, V(BB), as

$$V(BB)_0 = \lim_{T\to\infty} \sum_{t=1}^{T} \frac{R_t}{(1+r)^t} = \frac{R_1}{(1+r)^1} + \frac{R_2}{(1+r)^2} + \cdots \quad (4)$$

If there is a surplus, costs are less than the revenues and so, $min(R_t, C_t') = C_t'$ and $max(R_t - C_t', 0) = S_t$. We define the value stemmed from lucrative periods, V(P), as

$$V(P)_0 = \frac{C_1'}{(1+r)^1} + \frac{C_2'}{(1+r)^2} + \cdots + \frac{S_1}{(1+r)^{1+k}} + \frac{S_2}{(1+r)^{2+k}} + \cdots \quad (5)$$

One can easily notice that $V(BB) > V(P)$ since for every period t it holds that

$$\frac{R_t}{(1+r)^t} = \frac{C_t'+S_t}{(1+r)^t} > \frac{C_t'}{(1+r)^t} + \frac{S_t}{(1+r)^{t+k}} \quad (6)$$

Bringing forward the excess cash flows S_t allows the manager to increase V(P) to V(BB). Namely, a 100% depletion of revenues in every period t increases the value of the club. This is in line with the (corrosive) time value of money, suggesting that a unit of currency spent today has a greater value than a unit of currency spent tomorrow. Moreover, Madden and Robinson (2012), formalizing the seminal work of Sloane (1971) on utility maximization for European football clubs, argue that a sports club that records profits implies social sub-optimality if the fan welfare is considered. In the same vein, Madden (2012) suggests that there is no credible welfare case in favor of profit maximization for sports clubs and that profit maximization fails to produce a socially valuable outcome.

Sports clubs are usually loss-generating entities (especially in Europe, as discussed earlier) and if there is a deficit, costs exceed revenues. Therefore, in such a case it holds that $min(R_t', C_t) = R_t'$ and $max(R_t' - C_t, 0) = 0$. We define the value stemmed from deficit periods, V(L), as

$$V(L)_0 = \frac{R_1'}{(1+r)^1} + \frac{R_2'}{(1+r)^2} + \cdots \quad (7)$$

It is obvious that V(BB) is greater than V(L) since $R_t = C_t > min(R_t', C_t) = R_t'$. So, for every period t it holds that

$$\frac{R_t}{(1+r)^t} = \frac{C_t}{(1+r)^t} > \frac{min(R_t', C_t)}{(1+r)^t} = \frac{R_t'}{(1+r)^t} \quad (8)$$

Equation (7) is equivalent to Equation (7′) as the present value of the $C_t - R_t'$ arguments is zero for $k \to \infty$

$$V(L)_0 = \frac{R_1'}{(1+r)^1} + \frac{R_2'}{(1+r)^2} + \cdots + \frac{C_1-R_1'}{(1+r)^{1+k}} + \frac{C_2-R_2'}{(1+r)^{2+k}} + \cdots \quad (7')$$

when the costs exceed revenues in every period and there is no capital increase from the club's stockholders then, equity crunches will take place (possibly leading to negative book equity values) and/or the debt may rise to finance the gap. It is not good for the club's survivability, let alone for valuation purposes and the club's welfare. Eventually sanctions and relegation are triggered, and the club's bankruptcy follows.

V(max) is achieved via balanced budgets. Namely, when revenues equal costs it holds that V(BB) = V(max). The higher the revenues, the higher the sports club's value. Therefore, given

balanced budgets, wise spending, and revenue maximization, the benevolent manager maximizes the sports club's value.

V(*max*) is like V(*P*) with one difference. With V(*P*), any positive net income, i.e. surplus, is penalized and it is discounted in the next periods. Instead of depleting current surpluses in the next periods like V(*P*), V(*max*) brings them forward increasing value and $k = 0$ is implied in Equation (5).

The intermediate case is when there are profits and losses over time. We define this value, V(*P&L*), and it is like V(*P*) with one difference. With V(*P&L*) or V(*L*), any uncovered negative net income is penalized and is not taken into consideration in the discounting process at all. Though, if there is no equity crunch despite negative net income, i.e. share capital increases offset the deficit, then, V(*P&L*) and V(*L*) equate V(*BB*) *ex post* as if there was a balanced budget *ex ante* [$k = 0$ in Equation (7′)]. Furthermore, V(*P*) could deteriorate to V(*P&L*) if surpluses are not depleted at all and kept for capital adequacy. This is the case where the club treats profits as losses and a large value in *k* parameter ($k \to \infty$) is implied in Equation (5). Alternatively in valuation terms, V(*P*) and V(*P&L*) converge to V(*L*) if the surpluses are depleted after many years since they were created and consequently, their present value tends to be zero.

We notice that:

$$V(max) = V(BB) > V(P) > V(P\&L) > V(L) \tag{9}$$

In Table 1 we demonstrate the hierarchy of V(*BB*), V(*P*), V(*P&L*) and V(*L*) in a simplified three-period setup, where $k = 1$ in Equation (3).

Table 1: Comparison of values

	Period 1	**Period 2**	**Period 3**
V(*max*) = V(*BB*)	$R = 20, C = 20$	$R = 19, C = 19$	$R = 23, C = 23$
	$20 \cdot (1+r)^{-1}$	$19 \cdot (1+r)^{-2}$	$23 \cdot (1+r)^{-3}$
V(*P*)	$R = 20, C = 18$	$R = 19, C = 16$	$R = 23, C = 19$
	$18 \cdot (1+r)^{-1}$ $+2 \cdot (1+r)^{-2}$	$16 \cdot (1+r)^{-2}$ $+3 \cdot (1+r)^{-3}$	$19 \cdot (1+r)^{-3}$ $+4 \cdot (1+r)^{-4}$
V(*P&L*)	$R = 18, C = 20$	$R = 16, C = 19$	$R = 23, C = 19$
	$18 \cdot (1+r)^{-1}$	$16 \cdot (1+r)^{-2}$	$19 \cdot (1+r)^{-3}$ $+4 \cdot (1+r)^{-4}$
V(*L*)	$R = 18, C = 20$	$R = 16, C = 19$	$R = 19, C = 23$
	$18 \cdot (1+r)^{-1}$	$16 \cdot (1+r)^{-2}$	$19 \cdot (1+r)^{-3}$

Notes: R is revenues, C is costs and r is the discount rate. We assume $k = 1$ and $T = 3$ in Equation (3) for ease of exposition.

To sum up, according to Equation (9), if there was a sports club generating profits every year then, it could not achieve maximum value because the current surpluses in each period are depleted in the long term. On the other hand, if there were uncovered losses every year then, the club would be forced to default for obvious reasons and its value would be ripped off. Regarding the intermediate case, i.e. if there were some periods with profits and some periods with losses then, this value would be between the two associated values and of course, smaller than the maximum value which is attained with a strategy of balanced budgets. So, value is maximized when the teams aim to balanced budgets, wise spending, and revenue maximization and as a result, the value is higher than when teams maximize profits or record losses.

In practice, adjusted revenues can be employed, and V(L) and V($P\&L$) could be equal to V(BB) *ex post* as mentioned earlier, V(P) could be equal to V(BB) *ex post*, as well as V(P) and V($P\&L$) could equate V(L) *ex post*. Namely, in the first two cases (the third case), the manager reduces (increases) "waste" in valuation terms. Suppose that in the researcher's examined period, including the available historical data for the club's financial statements, the overall net income is negative [either it is negative for all periods, V(L), or it alternates between positive and negative values, V($P\&L$)] and covered by share capital increases. In such a case of balanced budgets *ex post*, these covered losses would augment revenues [$k = 0$ in Equation (7′)] in the valuation process and it would be as if there was balanced budget *ex ante*.[18] Also, another case of balanced budgets *ex post*, in practice, is when the surpluses are depleted and the overall net income approaches zero over of a fixed time period and, therefore, V(P) equals V(BB) *ex post* [$k = 0$ in Equation (5)]. On the other hand, if the overall net income is positive in the sample under consideration [either it is positive for all periods, V(P), or it alternates between positive and negative values, V($P\&L$)], then, the surplus would reduce revenues [$k \to \infty$ in Equation (5)] in the valuation procedure and profits are treated as losses.

In the case of dividends distribution to shareholders, our proposed valuation model is still applicable but there must first be a penalty on the employed revenues or costs, depending on whether we have profits or losses, in Equation (3). Particularly, we could subtract dividends as they reduce the available financial resources for running the club (e.g. to pay wages and other operating expenses). However, if there are share capital increases then, the covered amount of dividends could be added back in the valuation process.

[18] One could argue that this is in line with the underlying basis for the UEFA financial sustainability regulations (ex "financial fair play break even" rules). According to these regulations, sports clubs should balance their costs with their revenues over a fixed time period, and any losses beyond the minimum allowable loss (€60 million over a 3-year accounting period) are required to be covered by equity injections to prevent the build-up of debts. Also, a cap is introduced on wages, transfers, and agents' fees as these categories are limited to 70% of total revenues.

Claim #3

A sports club can be seen as a "quasi firm".

Reasoning:

In a seminal work, Neale (1964) reasons that teams are unconventional firms in output terms. He discusses the nature of firms in sports, defining the sporting firm with respect to the mixture of its peculiar output. He argues that professional teams are indeed in a peculiar position in relation to firms in a competitive market. Unlike typical firms that desire a monopoly status to maximize profits, monopoly for sports clubs is a total disaster in financial terms. Specifically, it is better for a sports club to compete with strong opponents rather than with weak ones because more sponsors get involved (and greater income is generated). Maybe it is good for a club to be the best club, monopolize its league and win every trophy in the short run, but in the medium-to-long term, such behavior is fatal to competition and sponsors investment, and a club should be just a good club and should avoid monopolizing its participating league(s). Finally, he characteristically argues that sporting teams should pray to Lord for being a good team, but not that good!

We have shown previously in "Claim #1" and "Claim #2" that a sports club, run by a benevolent manager, differentiates from an ordinary firm, in terms of its objectives, since it:

i. aims to balance budgets,
ii. does not create value through dividend policy as dividends may erode value,
iii. maximizes the sports club's welfare and revenues rather than profits, and
iv. may treat profits (losses) as if they were losses (profits).

On the other hand, a sports club maintains the maximization behavior and the aversion in losses. To sum up, sports clubs exhibit both similarities and differences with conventional firms, in both terms of output and objectives. Consequently, we consider a sports club as a hybrid firm, or a "quasi firm".[19]

3 Discussion and managerial implications

In the previous section we established a model of fundamental valuation for the special case of sports clubs under the framework of discounted cash flows, always keeping in mind the maximization of the sports club's welfare.

[19] However, what if a club is a profit maximizer, like American professional sports teams? Or, what if a (listed) club pays dividends to its shareholders (see e.g. Manchester United.)? Does it mean that these clubs are ordinary firms? The answer is negative, regardless of recording profits or losses, or regardless of paying dividends. It has already been argued about 60 years ago (see Neale, 1964) that sporting teams fundamentally differ from standard firms, because sport clubs are in a peculiar position due to the complex nature of their output.

The proposed estimation method for a sports club's value, incorporating (adjusted) revenues, could be seen as the fair price for a *whole takeover* of the club; this is the "club's total value", *CTV*, or the total enterprise value. Such an extreme action may include rebranding the acquired club, i.e. a new name, new emblem and new colors are adopted. Alternatively, *CTV* is the theoretical value of the club to be distributed to the associated fans and registered members, after their approval of the full takeover of the club and commitment to support the newly established club. Even if this is rather an unrealistic, but not an impossible scenario since several mergers have happened, *CTV* could be seen as a measure of the total money-generating ability of a sports club.

The fundamental part of *CTV* stems from the club's fans. As we stated in the previous section, fans are the "alpha and the omega" of a sports club and their crucial contribution to *CTV* depicts the club's core value. This is the value of the brand name, *BNV*, or the fundamental value, of the club. Alternatively, *CTV* can be treated as a conditional value on *BNV*.

$$CTV_0 = \begin{cases} \lim\limits_{T\to\infty} \left[\sum_{t=1}^{T} \frac{R_t}{(1+r)^t} \right] - X_0 \,, & BNV_0 > 0 \\ 0 \,, & \text{otherwise} \end{cases} \tag{10}$$

$$BNV_0 = max \left\{ \lim_{T\to\infty} \left[\sum_{t=1}^{T} \frac{a_0 \cdot R_t}{(1+r)^t} \right] - X_0 \,, \;\; 0 \right\} \tag{11}$$

where, X_0 is a penalizing term defined as the absolute value of negative book equity values, CTV_0 is the current club's total value, BNV_0 is the current value of the brand name[20], and a_0 represents the share of revenues related to the club's brand name with respect to total revenues.

The brand-name value is an asset for the shareholders and the starting point of negotiations for a change of the major shareholder, where the final price of the deal could be agreed after applying a *discount* or a *premium* on the fundamental value of the club. Namely, the brand-name value could be seen as the asking price.[21] Furthermore, the club's fundamental value stems from the fans-based revenues, i.e. money earned from the fans of the club. These are the matchday revenues including money earned from selling tickets (single or seasonal) and memberships, plus money earned from (1) selling material and clothing from the club's official stores or any other licensed consumer product with the club's name and emblem on it, and (2)

[20] Coates et al. (2017) propose an alternative measure of brand strength based on a football club's away attendance, and then examine determinants of this measure focusing on the Russian Premier League.

[21] Bertsatos and Sapountzoglou (2022) employed the proposed valuation model and their asking price for *Chelsea FC*, based on the fundamental brand-name value, was verified. More details are in the next section.

food and beverage from cafes inside or near the stadium. However, poor financial management should be penalized and in such a case, the brand-name value is reduced by X_0.

In the same spirit as the value of the brand name, the broadcasting component of *CTV* stems from the stream of broadcasting revenues. That is, money earned from TV and internet rights (e.g. pay per view or bundles). Also, the commercial element of *CTV* is due to advertisements, sponsorships and promoting brand names of sponsors firms. Moreover, revenues from selling players to other clubs or money prizes from domestic or international tournaments can be a special component of *CTV*. Namely given a positive brand-name value, *CTV* can be decomposed to j components and these associated values are estimated with Equation (4), where $R_{j,t}$ represents the revenue source j. Therefore, the sum of the brand-name value, *BNV*, at period m and the values, V_j, of the j-th revenues streams at period m constitutes the corresponding club's total value *CTV*.

$$CTV_m = \begin{cases} BNV_m + \sum_{j=1}^{J} V_{j,m} \ , \ BNV_m > 0 \\ 0 \ , \ \text{otherwise} \end{cases} \tag{12}$$

where, $V_{j,m}$ is calculated from Equation (4) as stated earlier and it is equal to 0 for a non-positive brand-name value.

Consequently, based on the club's financial data, a revenues decomposition –related to the stadium's money-generating ability– could help us to estimate the fundamental value of the sports club's owned home ground. This involves the calculation of the $x_j\%$ contribution of the j-th revenue source, where $j = 1,2,\dots,J$, that are related to the stadium, e.g. $x_1\%$ from revenue stream 1, $x_2\%$ from revenue stream 2, …, and $x_J\%$ from revenue stream j. Therefore, given an owned stadium, the stadium's value is a subset of CTV, and the complementary value is not related to the stadium's money-generating ability.

$$OSV_0 = \begin{cases} \lim_{T\to\infty} \left[\sum_{j=1}^{J} x_j\% \cdot \sum_{t=1}^{T} \frac{R_{j,t}}{(1+r)^t} \right] - X_0 \ , \ BNV_0 > 0 \\ 0 \ , \ \text{otherwise} \end{cases} \tag{13}$$

where, OSV_0 is the owned stadium's value at period 0.

However, if the club rents the stadium, or has signed a long-run leasing agreement with the owner (e.g. the state), negotiations for the acquisition of the stadium could be based on our suggested methodology. For example, the fair price could be a function of three elements: (1) the stadium's present value as if it was owned by the club, (2) the future value of past rental payments and (3) the future value of the total construction cost of the stadium.

The proposed framework of fundamental valuation could be useful to clubs' managers:

i. during negotiations for a hypothetical acquisition by determining the final offer, or the asking price, that brings closure to the deal from a fundamental perspective,
ii. for assessing fundamental metrics or fundamental key performance indicators (KPIs) of managerial efficiency, and maximizing the club's value,
iii. for assessing the value of the club's stadium in fundamental terms[22],
iv. for signing contracts with the club's grand sponsor, which may request for its name to be placed in the club's jersey or to be added next to the club's name, or for designing contracts when hiring athletes for the club's squad,
v. for making deals with retailers or wholesalers by authorizing them to sell their products with the club's name and emblem on them,
vi. for understanding why an athlete might reject a high contract offered by a low brand-name and/or high-budget club, and ultimately come to an agreement with a higher brand-name and/or low-budget club at a lower contract. The incentives of the athlete in this case are driven by the reputation and brand name of the club, and not by the money offered.

4 Empirical application

In this section, we present some basic financial variables of Panathinaikos FC. Next, we continue with the valuation part and finally, a hypothetical acquisition is examined along with assessing managerial efficiency from the point of view of fundamental analysis.

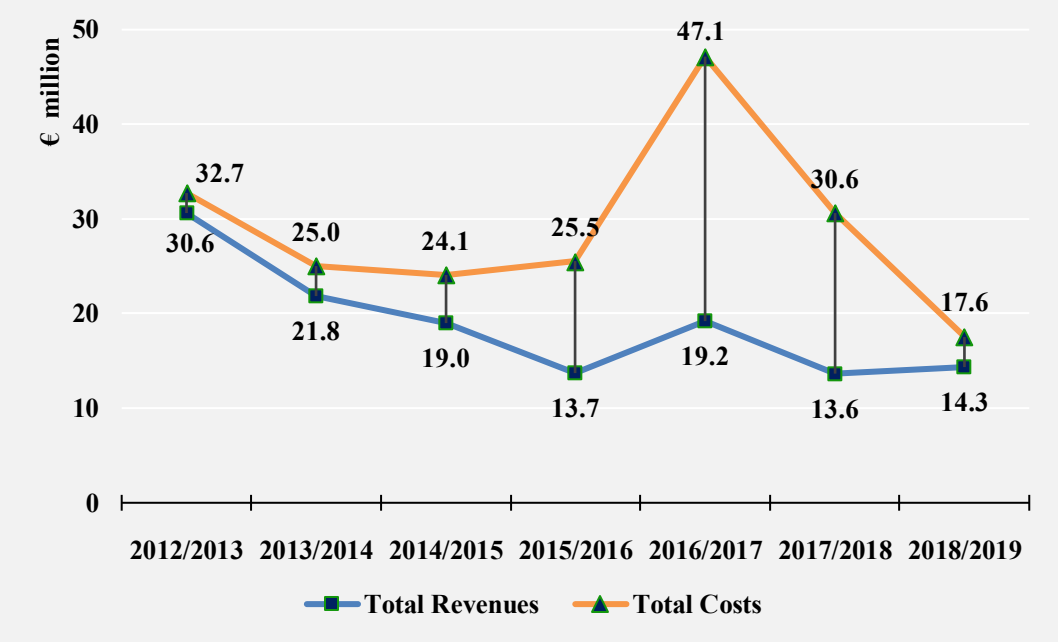
The multisport club of *Panathinaikos*, whose name means "*of all Athens*", was founded by *Georgios Kalafatis* on 3 February 1908 and the first stadium of the club was located at Patission Avenue, where the main building of *Athens University of Economics and Business* is located today[23]. We downloaded financial statements of the club from its official website, and we have annual data for seven periods starting from the season 2012/2013 up to the season 2018/2019. Also, in the first and the last season of our sample Panathinaikos FC rented the OAKA stadium to host the home matches. In the rest periods Panathinaikos FC used its iconic owned arena, Apostolos Nikolaidis Stadium or Leoforos Alexanadras Stadium (known as *Leoforos*), in the heart of Athens at Alexandra's Avenue for the home matches.[24]

[22] The suggested valuation model for sports stadia could complement or enrich literature on real estate valuation.

[23] Source: AUEB, https://www.aueb.gr/el/100/facts-archive/oikopedo-prin-htistei-kentriko-ktirio-toy-opa

[24] This is a key point for the valuation of Leoforos since it is improper to include periods, where OAKA hosted the home matches of the club.

Figure 1: Time evolution of total revenues and total costs (in € million)



Notes: Positive extraordinary items are added to total revenues and the absolute value of the negative extraordinary items is added to the absolute value of total costs. Corporate-tax payments increase total costs, while tax refunds are deducted from total costs. The accounting data are from the financial statements of Panathinaikos FC.

Total revenues for all the examined periods are on average €18.9 million, while the respective average total costs are €28.9 million. Figure 1 shows the annual values of total revenues and total costs of the club. In aggregate level, total revenues and total costs are almost €132.3 million and €202.6 million, respectively, leading to €70.3 million cumulative losses. Next, Table 2 shows the capital increases from season 2013/2014 to season 2018/2019 according to the Clean Surplus Accounting (CSA) relationship.

$$BVE_t = BVE_{t-1} + NI_t - DIV_t + W_t \quad (14)$$

where, BVE_t is the current book value of equity, BVE_{t-1} is the lagged-once book value of equity, NI_t is the current net income, DIV_t is the dividends paid and W_t corresponds to either net capital increases if it is positive or to net share repurchases if it is negative. During the period under consideration for our empirical application, share capital increases took place, and no dividends were paid. Table 2 shows the components of the CSA relationship.

Table 2: Components of the clean surplus accounting relationship (in €)

	2011/2012	2012/2013	2013/2014	2014/2015	2015/2016	2016/2017	2017/2018	2018/2019
BVE	-12,549,099	-8,604,159	-6,470,015	-4,266,816	-4,477,083	-24,406,051	-28,853,220	-14,125,767
NI	-	-2,073,781	-3,121,593	-5,135,993	-11,775,267	-27,931,131	-16,992,815	-3,228,271
W	-	6,018,721	5,255,737	7,339,192	11,565,000	8,002,163	12,545,646	17,955,724
C.W	-	6,018,721	11,274,458	18,613,650	30,178,650	38,180,813	50,726,459	68,682,183

Notes: The values of book value of equity and net income are obtained from the financial statements of Panathinaikos FC, while the net share capital increases (*W*) are calculated from the Clean Surplus Accounting Relationship [see Equation (14)]. *C.W* denotes the cumulative share capital increases. We highlight with blue font the highest absolute values of book value of equity, net income, and capital increases.

All periods in our sample suffer from negative net income, indicating inefficient allocation of financial resources. Despite the large and successive capital increases (see the penultimate row of Table 2), which sum up to €68.7 million from 2011/2012 to 2018/2019 with an average of €9.8 million per annum, we observe that the book value of equity persists to be negative for all the examined periods (book debt is higher than book assets in the balance sheets).[25, 26] This is clearly a first sign of poor financial management (accompanied by a poor match performance) leading to high debt burden and heightened cumulative losses, whose peak was the imposition of a 3-year ban to Panathinaikos FC by UEFA from participating in European tournaments for the seasons 2018/2019 to 2020/2021. However, analyzing the causes of the poor financial (and match) performance of the club is out of scope of this paper. Later in this section, we quantify the degree of managerial efficiency on a fundamental basis.

[25] Over the same period (2011/2012 to 2018/2019), book value of assets shrank to €14.7 million from €25.0 million, and book value of debt reduced to €28.8 million from €37.5 million. As a result, book value of equity further deteriorated from -12.5 million euros to -14.1 million euros.

[26] We should note that the balance sheets of sports clubs do not fully reflect all the assets. For example, the ongoing value of their supporter base and that of home-grown players are unrecorded assets, the access to lucrative competitions, and the book value of players registration does not reflect the current market value of these players (lack of fair value accounting). As a result, the book value of equity is a measure of relative balance sheet health (see e.g. UEFA, 2023) for sports clubs and is generally lower than should be.

Regarding the valuation part, we use either the average or the median of the required variables. In Equation (15) we note that revenues *R* and the growth rate *g* are value enhancers, while the discount rate *r* is corrosive in valuation terms.

$$V = h\left(\underset{R,}{+}\ \underset{g,}{+}\ \underset{r}{-}\right) \tag{15}$$

We move in a conservative way and specifically, we employ the lower of the two values (average or median) for the revenues and the growth rate, and the greater of these two values for the discount rate.

Since net income remains negative for the sample period under consideration, only revenues will be employed in the valuation procedure. Even if share capital increases covered the accumulated losses, the covered losses would not increase revenues in the valuation process due to the persistent negative book equity values.

Valuation of Panathinaikos FC

Equations (16) and (17) contain the formulas for estimating the brand-name value and total club's value of Panathinaikos FC, respectively.

$$BNV_0 = max\left\{\lim_{T\to\infty}\sum_{t=1}^{T}\left[(a_0 \cdot R_0)\cdot\prod_{j=1}^{t}\frac{(1+g_j)^j}{(1+r_j)^j}\right] - X_0,\ 0\right\} \tag{16}$$

$$CTV_0 = \begin{cases}\lim_{T\to\infty}\sum_{t=1}^{T}\left[R_0 \cdot \prod_{j=1}^{t}\frac{(1+g_j)^j}{(1+r_j)^j}\right] - X_0,\ BNV_0 > 0\\ 0,\ otherwise\end{cases} \tag{17}$$

where, R_0 is current revenues, g_j is the one-period growth rate at period *j*, r_j is the one-period discount rate at period *j*, a_0 represents the fans-based revenues in relation to total revenues, and X_0 is a penalizing term for negative equity values as discussed in Equations (10) and (11).

We perform stochastic Monte Carlo simulations to obtain the whole distribution of value rather than employing a point estimate of the valuation formula.[27, 28] To be more specific, we employ $N = 10{,}000$ replications and a value of $T = 300$ years. Further, we assume that:

[27] Glasserman (2003) and Agresti (2018), among others, provide details on Monte Carlo simulations and statistical methods.

[28] Damodaran is fond of the use of simulations in the valuation process; see indicatively https://aswathdamodaran.blogspot.com/2016/05/dcf-myth-32-if-you-don-look-its-not.html and https://aswathdamodaran.blogspot.com/2018/09/amazon-and-apple-at-trillion-follow-up.html

i. R_0 follows a normal distribution with the historical mean of revenues and the historical standard deviation of revenues of the club over the examined period (2012/2013 to 2018/2019). That is, $R_0 \sim N(\mu_R, \sigma_R) = N(18.90,\ 6.07)$ with a minimum value at 12.96, which is the 95% of the minimum value of historical revenues. For Leoforos valuation (2013/2014 to 2017/2018), the corresponding four numbers for the simulated revenues are 17.48, 3.64, 12.96 and 95%.

ii. $g \sim N(1.195\%,\ 1.195\%)$, i.e. the growth rate of the simulated revenues follows a normal distribution with the historical mean of the growth rate of annual GDP of Greece (source: ELSTAT, seasonally unadjusted nominal GDP) for the period 2002-2019 as both the average and the standard deviation of the growth rate of simulated revenues. For Leoforos valuation , the respective numbers are 1.145% and 1.145%.

iii. $r \sim N(4.613\%,\ 0.930\%)$, i.e. the discount rate of simulated revenues follows a normal distribution with a mean equal to the historical monthly mean of the annual interest rate of new loans, greater than €1 million, granted by Greek banks covering the period 2002:M09-2019:M12, and a standard deviation equal to the corresponding historical standard deviation of these interest rates on new loans (source: Bank of Greece). Also, the minimum value of the discount rate is the historical minimum value of these interest rates of bank loans, i.e. 2.960%. For the valuation of Leoforos the respective numbers are 4.710% (historical median of interest rates), 0.922% and 2.960%.

iv. a_0 is the ratio of the aggregate fans-based revenues (defined in Section 3) with the aggregate total revenues for the period 2012/2013 to 2018/2019 and equals 20.653%.

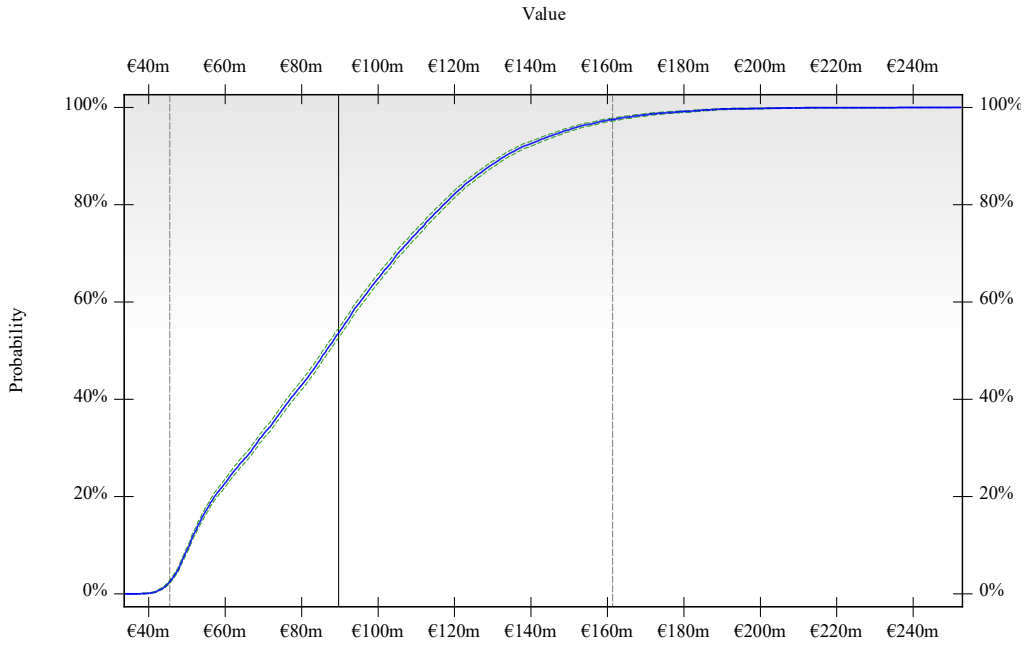
Regarding the parameters as we stated earlier [see Equation (15)], we have used either the average or the median value such that we underestimate positive-value drivers and overestimate negative-value drivers. For internal consistency in our case, we should also have used the average or the median of the negative book equity values as the penalizing term X_0. However, to remain conservative in our calculations we employ the maximum value of negative book value of equity, in absolute terms, over the period 2012/2013 to 2018/2019 for all the valuation procedures. Therefore, we set X_0 equal to €28.85 million (see Table 2).

According to the simulations, the average predicted value of brand name of Panathinaikos FC is €89.6 million (see vertical solid line at Figure 2) for the fiscal year ending June 2019.[29] We find that the value ranges from about €36 million to €251 million, while the 95% confidence interval, consisted of the 2.5th and 97.5th empirical percentiles, lies approximately between €45.5 million and €161.4 million (see vertical dotted lines at Figure 2). Regarding the club's

[29] Panathinaikos FC, and most football clubs, have a July-to-June fiscal year (FY), i.e. starting on 1 July of year t and ending on 30 June of year $t + 1$.

total value *CTV*, it turns out that the average predicted value is around €544.9 million, and the 95% confidence interval is between €331.0 million and €892.3 million.[30]

Figure 2: Empirical cumulative distribution function of the value of brand name



Notes: We used 10,000 replications for the generation of the distribution of the brand-name value of Panathinaikos FC. Left and right axes show the cumulative probability, and the top and bottom axes show the value. We used EViews 9 for the construction of the empirical cumulative distribution function and the 95% confidence intervals are based on the "Wilson interval methodology" (see Wilson, 1927, and Brown et al., 2001).

Valuation of Leoforos

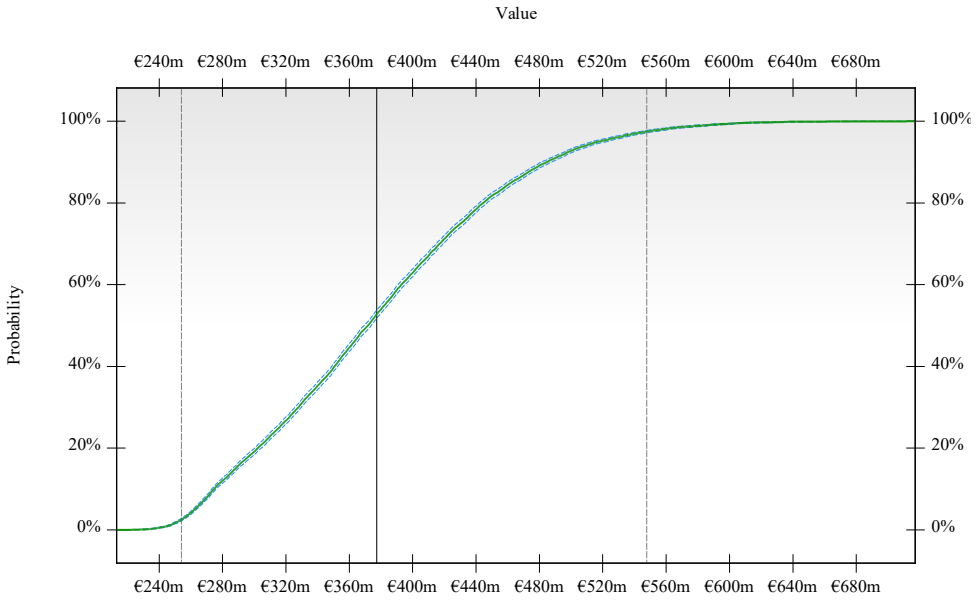
Next, we estimate the fundamental value of Leoforos based on Equation (18). To this purpose, we focus on period 2013/2014 to 2017/2018, where Panathinaikos FC used Leoforos for the home matches.

$$V(Leoforos)_0 = b_0 \cdot \sum_{t=1}^{T}\left[R_0 \cdot \prod_{j=1}^{t} \frac{(1+g_j)^j}{(1+r_j)^j} \right] - X_0 \qquad (18)$$

[30] To find the updated values, one could calculate the future value of the estimated values. For an extra degree of conservatism, the minimum value between [1] the inflation rate in Greece between June 2019 and the month of interest, and [2] 2.960% (i.e., the minimum value of one-period interest rate employed in the simulations) compounded over the same period, is suggested as the future value interest factor.

where, R_0, g, r, T, X_0 are defined earlier and $b_0 = 80.779\%$ is the share of stadium's related aggregate revenues to aggregate total revenues for the period 2013/2014 – 2017/2018. For b_0 we assume that (1) 100% of fans-based and broadcasting revenues contribute to the stadium's value, (2) 75% of commercial revenues (e.g. advertisements and sponsorships) contribute to the stadium's value, and (3) 50% of the rest income sources (UEFA solidarity fund and other operating revenues).

Figure 3: Empirical cumulative distribution function of Leoforos value



Notes: We used 10,000 replications for the generation of the distribution of Leoforos value. Left and right axes show the cumulative probability, and the top and bottom axes show the value. We used EViews 9 for the construction of the empirical cumulative distribution function and the 95% confidence intervals are based on the "Wilson interval methodology" (see Wilson, 1927, and Brown et al., 2001).

Simulations suggest that the average estimated fundamental value, for the fiscal year ending June 2018, of Panathinaikos FC home ground, Leoforos, is €377.3 million (see the vertical solid line at Figure 3). To the best of our knowledge, this is the first valuation of Leoforos that is publicly available. Leoforos value ranges between €254.1 million and €547.8 million (see the vertical dotted lines at Figure 3) with 95% probability. Alternatively, there is a 97.5% probability that it exceeds €254 million, and a 2.5% probability that the value does not exceed €548 million. Also, the minimum and maximum values are €218.5 million and €711.9 million.

An acquisition of Leoforos by a third party could imply, on average, a €377.3 million increase in Panathinaikos bank account (June 2018, current prices). The valuation of Leoforos

is very crucial, especially when it comes to the double regeneration project. This much-discussed project includes, among other things, the demolition of Leoforos, and the construction of a municipality football stadium, located in the area of Votanikos-Elaionas, which will be leased to Panathinaikos from the Municipality of Athens. According to the official announcement of Municipality of Athens on November 2020 (see here), the initially expected cost of the whole project is €163.5 million and Leoforos value is about €40 million. Therefore, combining this value of €40 million and the fundamental value of Leoforos of €377.3 million, it appears that approximately a 90% discount on Leoforos value is applied by the Municipality of Athens.

Employing the cumulative inflation 16.80% in the 2018-2024 period in Greece (CPI was 101.01 in 2018 and 117.98 in 2024 according to ELSTAT), the future value in 2024 of the fundamental value of Leoforos is about €440.70 million. Also, the future value in 2024 of the cost of the whole project is equal to €202.25 million given the soaring price index of construction cost of new buildings (the index was 96.86 in 2020 and 119.82 in 2024 according to ELSTAT, amounting to a cumulative 23.70% increase). The difference between these future values amounts to €238.45 million, and as a result, a policy implication arises: Financial losses of €238.45 million could be implied for Panathinaikos ("one side of the coin") and financial gains of €238.45 million for the rest stakeholders of the project ("the other side of the same coin"). In other words, the double regeneration project, when completed, could entail financial losses (gains) of more than €238 million for Panathinaikos (the other parties of the project).

A hypothetical acquisition

Now we are examining a hypothetical scenario involving a change in the club's management. Suppose that an entrepreneur or a group of companies wants to acquire Panathinaikos FC. How much is the new stockholder (bidder) willing to pay the existing stockholder (target) to close the deal? To put it differently, what is the asking price for putting Panathinaikos FC up for sale? We answer this question utilizing the *status-quo* value of the brand name on a fundamental basis, i.e. the value of the brand name under the existing management (see "Valuation of Panathinaikos FC" subsection above).

Based on our previous findings and given that no special events – related to the hypothetical acquisition – exist, the fundamentally justified asking price for selling Panathinaikos FC on behalf of the existing management, for the financial year ending June 2019, is €89.6 million, on average, while the 95% confidence interval ranges from €45.5 million to €161.4 million.

On the other hand, let one special event –related to the hypothetical acquisition– exist: suppose that the major stockholder of Panathinaikos FC, Ioannis Alafouzos, follows Roman Abramovich's rationale with Chelsea FC (see Bertsatos and Sapountzoglou, 2022, for more details) and puts the club up for sale without asking for his money spent in the club to be recouped. In such a case, the average fundamental asking price for selling the club would reduce

to €20.9 (= €89.6 − €68.7) million as the cumulative share capital increases over the seasons 2012/2013 – 2018/2019 are near €68.7 million (see last row of Table 2). Moreover, the 95% confidence interval for the fundamental asking price in this case would range from €0 [= max(€45.5 − €68.7, €0)] to €92.7 (= €161.4 − €68.7) million.

However, the final price that brings closure to the deal with either a *discount* or a *premium* on the average fundamental asking price depends on the negotiating power of the bidder and the target, and on the willingness/pressure of the fans for a change in the existing club's management. In other words, this may imply that the final price tag tends to the lower, or to the upper bound (if "ego and pride premia" or "ego factor" are triggered, see e.g. Football Benchmark, 2022; Damodaran, 2014; Vine, 2004) of the predicted confidence interval of the *status-quo* value of the brand name.

At this point we should note that Bertsatos and Sapountzoglou (2022) employed this valuation model in the case of Chelsea FC. Based on data up to 2020/2021 season and their estimate of almost £4.5 billion for the average fundamental brand-name value, the predicted 95% confidence interval of asking prices was verified by the final price –£4.25 billion– that brought closure to the deal. On the other hand, the prediction of £1.6 billion by specialists and media (see e.g. here) turned out to be wrong, whilst Chelsea's brand value was £725 (742) million in 2022 (2023) according to *Brand Finance* (see here) and $504 ($480) million in 2024 (2023) based on *Forbes* (see here).

Next, we propose three fundamental key performance indicators to evaluate the existing management's performance for Panathinaikos FC: the value of control (VOC), the fundamental growth rate (FGR), and the fundamental managerial efficiency (FME). First, we estimate the optimal value of the brand name. The optimal value of the brand name implies management that adheres to the core principles and claims discussed earlier, i.e. balanced budgets (*ex post*) and efficient allocation of revenues. In other words, the optimal brand-name value is the value of the brand name as if the club was optimally run. In such a case, we take into account the augmented revenues and cancel the penalizing term X_0 in the valuation procedure of the optimal brand-name value. Consequently, the historical revenues are augmented by the amount of any losses, as under optimal management any losses are covered by share capital increases. These revenues are the modified or counterfactual revenues, which the optimal value of brand name of Panathinaikos FC is based on.[31]

[31] Since optimal management pins down to zero the penalizing term and covers any losses with share capital increases then, modified or counterfactual revenues are employed in the valuation process. As a result, both the club's total value (CTV) and owned stadium value (OSV) increase relative to *status quo* values.

As mentioned earlier we run simulations with $N = 10{,}000$ replications for the valuation of brand name according to Equation (16). For each replication $n = 1, 2, \dots, N$, we calculate the corresponding optimal value of brand name BNV_{opt} according to Equation (19).

$$BNV_{opt,n} = \frac{BNV_{stq,n} + X_0}{a_0 \cdot \bar{R} + \bar{Z} \cdot \mathbf{1}\{\text{optimal management}\}} \cdot (a_0 \cdot \bar{R} + \bar{Z}) \tag{19}$$

First, we cancel the penalizing term $X_0 = €28.85$ million, due to the assumption of optimal management, by adding it back to the *status-quo* value of brand name $BNV_{stq,n}$ [see Equation (16)]. Next, we divide that outcome with the sum of (1) the ratio $a_0 = 20.653\%$ of the historical aggregate fans-based revenues with the historical aggregate revenues times the historical average of the revenues $\bar{R} = €18.90$ million, and (2) the historical average of the losses $\bar{Z} = €10.04$ million –that would be covered under optimal management– times an indicator function that takes the value of 1 for optimal management and 0 otherwise. Finally, multiplying the last outcome with the counterfactual revenues, i.e. the sum of the historical average of revenues $\bar{R}$ times the ratio a_0, and the historical average of losses $\bar{Z}$, gives us the brand-name value as if the football team of Panathinaikos was optimally run. In other words, in our case for the optimal value of brand name of Panathinaikos FC, the term $BNV_{stq,n} + X_0$ is first deleveraged with $a_0 \cdot \bar{R}$ and then leveraged with $a_0 \cdot \bar{R} + \bar{Z}$.

Based on our simulations, we find that for the financial year ending June 2019, the optimal value of the brand name is on average €423.3 million, and the 95% confidence interval ranges from €265.5 million to €679.6 million.

Equation (20) shows that the value of control equals the optimal value of brand name BNV_{opt} reduced by the *status-quo* value of brand name BNV_{stq}. Alternatively, value of control (VOC) is the value added under optimal management of the club while keeping other things constant. Also, the higher the value of control we find, the less efficient the existing management is.

$$Value\ of\ control \equiv VOC = BNV_{opt} - BNV_{stq} \tag{20}$$

We find that the average value of control of Panathinaikos FC equals €333.6 million, while the 95% confidence interval ranges from €220.0 million to €518.2 million.

Given the optimal and *status-quo* values of brand name for each replication n, we estimate the fundamental growth rate (FGR) of Panathinaikos FC brand name, as shown by Equation (21). The range of *FGR* is $[0, \infty)$ by definition. We find that *FGR* equals 387.9% on average, while the 95% confidence interval ranges from 321.1% to 483.8%. This is evident of strong financial incentives in acquiring and optimally running Panathinaikos FC.

$$FGR_n = \frac{BNV_{opt,n}}{BNV_{stq,n}} - 1 \quad (21)$$

Finally, we estimate fundamental managerial efficiency (FME) based on the estimates of *status-quo* value of brand name and optimal brand-name value from the N replications. Specifically, it is the *status-quo* value of brand name over the optimal value of brand name.

$$FME_n = \frac{BNV_{stq,n}}{BNV_{opt,n}} \quad (22)$$

By construction *FME* lies in the interval $(0{,}100\%]$. In fact, under optimal management there is a zero-penalizing term X_0 and therefore, the *status-quo* and optimal brand-name values are identical [see Equation (19)], whilst for positive values of X_0 the optimal value of brand name exceeds the *status-quo* value [see Equation (16)].

$$FME_n = \frac{1}{1+FGR_n} \quad (23)$$

Combining Equations (21) and (22), reveals that *FME* equals the inverse of the sum of 1 and *FGR*. We argue that the fundamental growth rate and managerial efficiency are two sides of the same coin. To put it differently, high (low) *FME* implies low (high) *FGR*, as well as decreased (increased) incentives for a bidder, in the sense that it is more (less) difficult to create value for a lucrative future acquisition by another bidder, keeping other things constant. Also, if the club under consideration is optimally run, i.e. there are balanced budgets (*ex post*) and efficient allocation of revenues is achieved, then the *status-quo* value of brand name equals the optimal brand-name value and as a result, *FGR* becomes 0 and *FME* equals 100%.

In our case, the average *FME* is found to be 20.7%, and the 95% confidence interval ranges from 17.1% to 23.7%, implying a low degree of fundamental managerial efficiency of the existing management and denoting that there is plenty of room for improvement in running the football team of Panathinaikos.

Table 3 visualizes and summarizes our findings in three panels based on fundamental analysis. Panel A shows the brand-name value and total value of Panathinaikos FC, while Panel B shows the value of Leoforos. In Panel C, one can find the suggested asking price for putting Panathinaikos FC up for sale and the value of control for a hypothetical acquisition, the optimal value of brand name of Panathinaikos FC as well as, the fundamental growth rate and fundamental managerial efficiency.

Table 3: Estimated fundamental values and metrics for Panathinaikos

Panel A, sample period 2012/2013 – 2018/2019. On the club's valuation	
Value of brand name, FY 2019	*€ 89.6 million*
	[45.5, 161.4] million
Total value, FY 2019	*€ 544.9 million*
	[331.0, 892.3] million
Panel B, sample period 2013/2014 – 2017/2018. On the stadium's valuation	
Leoforos value, FY 2018	*€ 377.3 million*
	[254.1, 547.8] million
Panel C, sample period 2012/2013 – 2018/2019. On a hypothetical acquisition	
Optimal value of brand name, FY 2019	*€ 423.3 million*
	[265.5, 679.6] million
Asking price, FY 2019	*€ 89.6 million*
	[45.5, 161.4] million
Value of Control, FY 2019	*€ 333.6 million*
	[220.0, 518.2] million
Fundamental growth rate, FY 2019	*387.90%*
	[321.1%, 483.8%]
Fundamental managerial efficiency, FY 2019	*20.70%*
	[17.1%, 23.7%]

Notes: Panel A displays the brand-name value and the total value of Panathinaikos FC, on average, for the "fiscal year ending June 2019" using data for the seasons 2012/2013 – 2018/2019, while Panel B shows the average value of Leoforos for the "fiscal year ending June 2018" using data for the seasons 2013/2014 – 2017/2018. Panel C presents, on average, the asking price for selling Panathinaikos FC and the value of control for a hypothetical acquisition, and the optimal value of brand name of Panathinaikos FC for the fiscal year ending June 2019. Also for the fiscal year ending June 2019, the fundamental growth rate and fundamental managerial efficiency, on average, are presented in Panel C using data for the seasons 2012/2013 – 2018/2019. The average values are shown in the first row of each cell. The numbers in brackets in the second row of each cell display the 95% confidence interval with the lower and upper bounds equaling, respectively, the 2.5th and 97.5th empirical percentiles.

Conclusions

In this paper we provide a framework for the maximization of the sports club's welfare and value. In such a case, balanced budgets are adopted and overpricing policies on tickets are avoided. We argue that a sports club should be treated as a "quasi firm" and propose a fundamental valuation model and fundamental financial metrics of managerial efficiency. According to our framework, the (benevolent) manager of a sports club aims to achieve:

i. balanced budgets,
ii. efficient allocation ("wise spending") of the financial resources, and
iii. revenue maximization.

In this way, the manager maximizes the sports club's welfare, and the sports club's value via discounting (adjusted) revenues, and increases the probability for the team to excel in match seasons and win titles. The maximum value could also be obtained under balanced budgets *ex post*, or under dividend payments, if deficits and dividends are covered by share capital increases and surpluses are depleted in the next periods (balanced budgets *ex post*). Furthermore, we demonstrate that a sports club's owned stadium constitutes, in valuation terms, a subset of the club's total value.

The proposed valuation model is applied to the Greek football team of *Panathinaikos* and its arena, *Leoforos*. We show for Panathinaikos FC that, on average, the team's total value is near €544.9 million and the brand-name value is around €89.6 million for the financial year ending June 2019, respectively, and that Leoforos value is on average about €377.3 million for the financial year ending June 2018. Given this fundamental estimate of Leoforos value –which is the first documented value that is publicly available as far as we are concerned– and the €40-million estimate for Leoforos value according to Municipality of Athens, one could claim that the Municipality of Athens underestimates Leoforos value approximately 90%. In fact, this is indicative of the adverse terms for Panathinaikos and its significantly weak negotiating power in the much-discussed double regeneration project, compared to the rest counterparties of the project. Instead, Panathinaikos could have had the upper hand in the negotiations for that project and could have demanded much better terms. Finally, a policy implication arises for the double regeneration project: The empirical findings, and the future values of the cost of the project and the fundamental value of Leoforos in 2024, suggest that there could be €238.45 million financial losses for Panathinaikos ("one side of the coin") and €238.45 million financial gains for the rest stakeholders of the project ("the other side of the same coin").

Our results provide overwhelming evidence of poor financial management for Panathinaikos FC as the average fundamental managerial efficiency is 20.7%, and of strong financial incentives for acquisition as the average fundamental growth rate of brand name is 387.9%. The value of control is on average €333.6 million indicating a large deviation of the

optimal value from *status-quo* value of brand name, and the fundamentally estimated asking price for selling Panathinaikos FC –under a hypothetical acquisition– is €89.6 million on average, for the financial year ending June 2019.

The proposed valuation model can be applied to a sports club in any team sport and can improve the decisions of the clubs' managers on various financial actions. This valuation model could help in the ranking of different sport clubs according to the (risk-adjusted) brand-name value and the growth rate of brand name and the degree of managerial efficiency from a fundamental financial point of view. It could also shed light during negotiations for takeovers by determining the final offer of the bidder (see e.g. Bertsatos and Sapountzoglou, 2022, for the case of *Chelsea FC*). A natural extension, and following the case of Chelsea, could be the *ex-post* examination of the sale of Manchester United FC and its fundamental valuation. Finally, another set of future directions could involve the application of the proposed valuation model to, for example, the top European football clubs participating in the UEFA Champions League or the most popular football teams at national level.

Declaration of conflict of interests

We declare that the views expressed are our own and do not necessarily represent the position of the institutions we are affiliated with. We also declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this paper.

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