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On the Role of Luck for the Concentration of Capital and Piketty's 'Fundamental Force of Divergence'

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We consider an overlapping generations version of a model suggested by Fargione, Lehmann and Polasky that allows us to show, by means of simulations, that randomness of the rate of return on capital, combined with inheritance of capital and consumption being a concave function of wealth may lead to an increasing concentration of capital. We can also show that the average rate of return being higher than the growth rate of aggregate income, r>g, does not necessarily lead to increasing concentration and that there are cases where concentration of capital does increase while the opposite inequality, g>r, holds.

Keywords: capital concentration, wealth distribution, Piketty, simulation, random returns

JEL Classifications: D31, E17, O43•

1 Introduction

Almost a century ago, Keynes (1936, p. 372) counted the 'arbitrary and inequitable distribution of wealth' as one of the 'outstanding faults of the economic society in which we live'. While Davies and Shorrocks (2000) in their survey article on the distribution of wealth could state that 'inequality has, on the whole, trended downwards in the twentieth century', in recent decades concern over rising concentration of wealth has again become one of the most troubling issues for economists such as Piketty (2014) or Stiglitz (2015a). In fact, while there is a rich empirical literature examining, both at a national level and globally, how the concentration of wealth has changed over time (e.g., Albers et al., 2022; Alvaredo et al., 2018; Blanchet & Toledano, 2023; Chatterjee et al., 2022; Zucman, 2019), and many differences between time periods and countries considered have been established, it may be regarded as a 'stylized fact' about the half century behind us that the concentration of wealth has been increasing considerably (Stiglitz 2016).

However, unlike most other stylized facts of economics, the increasing concentration of wealth does not seem to have a 'standard', let alone agreed-upon, explanation among

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economists. In fact, unlike the distribution of income, the distribution of wealth has become a significant topic among economists only in recent decades. E.g., the 990-page textbook on ‘Modern Economic Growth’ by Acemoglu (2009) starts out with basic facts about cross-country income differences – but hardly even mentions the issue of wealth distribution. To be sure, Piketty’s attempt to identify ‘the fundamental laws of capitalism’ has not only engendered much public attention to the changes in wealth distribution, but it has also sparked off a lively debate within the economics profession about their causes. While, e.g., Solow (2014) argued that ‘Thomas Piketty is right’, Acemoglu and Robinson (2015) insist that ‘the quest for general laws of capitalism is misguided because it ignores the key forces shaping how an economy functions: the endogenous evolution of technology and of the institutions and the political equilibrium that influence not only technology but also how markets function and how the gains from various different economic arrangements are distributed’.

From a methodological point of view, Piketty’s ‘fundamental laws’ are in fact somewhat strange[1] – but so is the suggestion that private ownership and inheritance of capital are somehow not among ‘the key forces shaping how an economy functions’. Much more interesting than Piketty’s two ‘fundamental laws’, however, is what he calls the ‘fundamental inequality’, namely, r>g. About this he claims that it is ‘the fundamental force for divergence’: ‘When the rate of return on capital exceeds the rate of growth of output and income, …, capitalism automatically generates arbitrary and unsustainable inequalities that radically undermine the meritocratic values on which democratic societies are based’ (Piketty, 2014, p. 1). From a traditional perspective on economic theorizing, one may deplore the absence of a clearly defined set of conditions that could be shown to imply Piketty’s claim[2], but the ‘fundamental force of divergence’ may at least be regarded as an interesting hypothesis about an important stylized fact – the increasing concentration of wealth.

Acemoglu’s above-mentioned textbook of growth theory also has a chapter (pp. 109 ff.) on ‘the fundamental determinants’ behind the models that populate the remainder of the book. Acemoglu’s ‘four fundamental causes’ are luck, geography, culture, and institutions – and he sees their investigation as important because the theory ‘without understanding the underlying driving forces, would be incomplete’ (p. 110). Completeness, however, is nothing that a theory can achieve anyway. It is quite obviously the case that Piketty’s account of capitalism in the

[1] Piketty’s first ‘law’ is an accounting identity, stating that α, the share of income from capital in aggregate income, must be equal to the product of r, the rate of return on capital, and β, the capital/income ratio. His second ‘law’, that the capital/income ratio equals the ratio s/g, where s is the savings rate and g the growth rate of income, is meant to ‘hold’ only in the long run. What this ‘long run’ is, is not explained by Piketty – an economist may think of the long run as jargon for a ‘steady state’. If the latter is defined as a situation where capital and income are growing with the same rate, the proposition that ‘β=s/g in the long run’ is again true by definition (cf. Krusell and Smith, 2015), but may not be terribly interesting - as we are all dead in the long run, as Keynes famously remarked.

[2] Piketty is in fact highly critical of ‘the childish passion for mathematics’ (p. 32) among economists.

21st century ignores many layers of forces and details that a fuller picture might provide. Nevertheless, it is an account that ties an important stylized fact to a set of institutions – among them private ownership of capital and inheritance thereof – that is sufficiently widespread to deserve analysis. A model that allows one to clearly derive the fact from the set of institutions may serve to better understand their connection. Obviously, no model can provide a 'complete' analysis, but neither can a 700 or 900 page book.

There are, of course, theoretical models that aim at elucidating the concentration of wealth. However, almost all such models focus on steady states (cf., e.g., the extensive reviews by De Nardi, 2015, or Benhabib and Bisin, 2018) and leave the process of wealth concentration unexplained. Some, of course, do aim at elucidating the forces behind the observed increase in wealth concentration. While certain modifications of the neoclassical growth model, such as Hiraguchi (2019), are 'consistent with Piketty's prediction that economic stagnation raises r−g and makes wealth distribution more unequal' (Hiraguchi, 2019, p. 481), the more ambitious set of models developed by Stiglitz (2015b, 2015c, 2015d, 2015e, 2016) aims at explaining increasing wealth concentration along with some other 'new stylized facts', such as a growing wealth-income ratio while rates of return are non-decreasing and average wages are stagnating. To do so, Stiglitz modifies the neo-classical growth model by distinguishing between wealth and capital, and explicitly acknowledging land (or real estate) as a non-produced asset. In Stiglitz's models 'the degree of inequality in the long-run equilibrium is *not* related to the difference between the growth of the economy and the return to capital' (Stiglitz 2016, p. 50).

In terms of methodology, however, both Hiraguchi's and Stiglitz's models are focussing on steady states. According to Stiglitz (2016, p. 4), 'we can best understand what has been happening as a shift from one equilibrium to another', and Hiraguchi (2019, 4. 484) explains that for 'the equilibrium path out of the steady state … wealth distribution is not analytically tractable'. However, it may well be that the observed increasing wealth concentration is not just the transition from one steady state to another. After all, steady states need not be stable and there are perfectly reasonable models where non-trivial steady states do not even exist. As growth seems to be driven by continuous and sometimes disruptive technological innovations and entrepreneurial activity, the very notion of a 'steady' state may be somewhat misleading. But this does not mean that analysis is utterly impossible. Even if one does not rely on the analytically nice steady-state methodology, one may well calculate – or "simulate" – the process of a changing wealth distribution numerically.

This is precisely what was done is an elegant short paper by Fargione, Lehman & Polasky (2011). They provide a simple dynamic model which shows that 'chance alone, combined with the deterministic effects of compounding returns, can lead to unlimited concentration of wealth'. Somewhat surprisingly, this insight has received little attention in the economic theory literature. To some extent this may be due to the fact that the model of Fargione et al. appeared before the 'Piketty hype', and that it appeared in a journal publishing mostly research in the natural sciences and medicine, but it may also have been overlooked by economists because the model is devoid of any explicit microeconomic ingredients such as labour, consumption, and individual optimizing behaviour[3].

In what follows, we therefore provide a more standard, discrete-time, 'overlapping generations' (OLG) version of the model of Fargione et al., where an increasing concentration of wealth can be derived from essentially four assumptions: (i) random rates of return on capital, (ii) the compounding of those returns, (iii) intergenerational transfers of capital ownership, and (iv) consumption being a concave function of wealth. Incidentally, our model also allows one to shed some light on Piketty's 'fundamental force of divergence'.

2 The Model

We assume that there is a fixed number n of dynasties, each of which consists of a sequence of generations. We do not specify a fixed lifetime for each generation, but assume that whenever a generation dies, there is a generation of heirs that inherits the capital of its parent generation. (An alternative interpretation of the model is as a variant of the OLG model with 'impure altruism' as discussed in, e.g., Acemoglu, 2009, 342 ff., where each generation lives for just two periods.)

We assume that at (the end of) time t, which we take to be measured in years, the wealth of the *i*-th dynasty is

$$v_{i,t} = w + \left(1 + r_{i,t}\right) b_{i,t-1}, \tag{1}$$

where w is the wage, assumed to be constant[4], $r_{i,t}$ is the rate of return, net of depreciation, on the capital of dynasty i in period t, and $b_{i,t-1}$ is the capital transferred – or inherited - within dynasty i from the previous period $t-1$. One can regard equation (1) as resulting from a simple linear production function, according to which an all-purpose commodity is produced from labour and capital, resulting in an income $y_{i,t} = w + r_{i,t} \cdot b_{i,t-1}$. Assuming that, in each period, each dynasty is endowed with one unit of labour which is fully used in technologically efficient production and measuring values in units of the all-purpose commodity, we can interpret w as labour productivity and $r_{i,t}$ as the productivity of the capital that dynasty i 'inherits' in year t from the previous year.

We further assume that, in each period, each dynasty decides how much of its wealth to consume in the current period and, thereby, how much to transfer to the next period. Consumption of dynasty i in period t will be denoted by $c_{i,t}$ and the capital it transfers from

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[3] Moreover, the model of Fargione et al. is framed in discrete time, but uses continuous compounding of returns. While this is, of course, logically ok, it is at least somewhat unusual in economics.

[4] This assumption will be modified in Section 6.

period t to $t+1$ by $b_{i,t}$. We also assume that preferences of all dynasties are identical. For the time being, we assume that the preferences of dynasty i in period t can be represented by a utility function of the form

$$u(c_{i,t}, b_{i,t}) = \log(c_{i,t}) + \mathrm{a}\sqrt{b_{i,t}} \tag{2}$$

While this is admittedly a rather special kind of utility function, and we will look at alternative assumptions in Section 5 below, the behavioural implications of (2) are quite reasonable for the present context. Maximizing utility subject to the budget constraint[5]

$$c_{i.t} + b_{i,t} \leq v_{i,t} \tag{3}$$

yields the consumption function

$$c_{i,t} = -\frac{2}{\mathrm{a}^2} + \frac{2}{\mathrm{a}}\sqrt{\frac{1}{\mathrm{a}^2} + v_{i,t}} \tag{4}$$

and the capital transfer function

$$b_{i,t} = \frac{2}{\mathrm{a}^2} + v_{i,t} - \frac{2}{\mathrm{a}}\sqrt{\frac{1}{\mathrm{a}^2} + v_{i,t}} \tag{5}$$

Thus, while both consumption and capital transfers are increasing with wealth, consumption is greater than the capital transfer for small values of wealth, but smaller for large values. This is illustrated in Figure 1 for the case of a=2

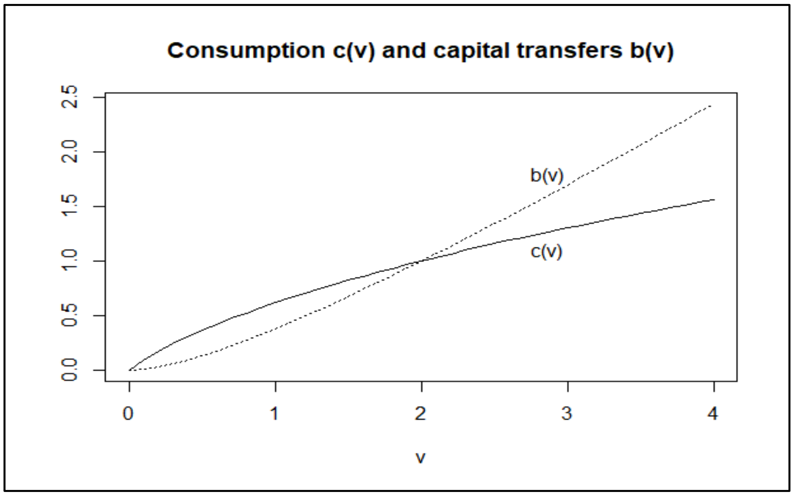
Combining these two equations with equation 1, one gets a difference equation for capital transfers:

$$b_{i,t} = w + (1 + r_{i,t})b_{i,t-1} + \frac{2}{a^2} - \frac{2}{a}\sqrt{\frac{1}{a^2} + w + (1 + r_{i,t})b_{i,t-1}} \tag{6}$$

Assuming also that every dynasty begins with the same capital endowment, $b_{i,1} = B$, this allows one to calculate the time path of capital holdings for each dynasty, if we also specify the values of B, w and the $r_{i,t}s$. We assume that the latter are random variables, reflecting the fact that investments are risky: picturing the all-purpose commodity of our model as corn, the return from current seed depends on factors like the weather conditions and may well be negative.

[5] The constraint (3) can, of course, be aggregated across dynasties to yield an aggregate resource constraint for period t. This is not essential for our analysis, however, as we do not explicitly consider exchange between dynasties.

Figure 1



More specifically, we assume that the $r_{i,t}s$ are derived from random variables which are independent and identically distributed according to a normal distribution with mean μ and standard deviation σ. However, to ensure that capital holdings are always non-negative and the root in equation (6) always stays within the reals, we truncate the normal distribution at the value -1. More precisely, we draw the $r_{i,t}s$ from a lower truncated normal distribution with lower bound -1, the parent of which has mean μ and standard deviation σ (cf. Burkardt, 2014, for details).

It seems hard to characterize the resulting time-paths in general terms, but it is easy to calculate alternative time-paths numerically, e.g., with the MATLAB software or the R language. We used R Version 4.2.3 (R Core Team, 2023) and its 'truncnorm' package (Mersmann et al., 2023).

3 A Baseline Case

It turns out that, for many combinations of parameter values, the Gini coefficient G(t) for the distribution of capital $b_{i,t}$ among a population of *n*=100,000 dynasties is essentially increasing over time. We consider a baseline case first. Figure 2 shows its time path of Gini Coefficients.

For this baseline set of parameter values, the average per-dynasty capital after 200 years is about 62, with more than 99% of dynasties having a capital of less than 200, as illustrated by Figure 3.

Figure 2

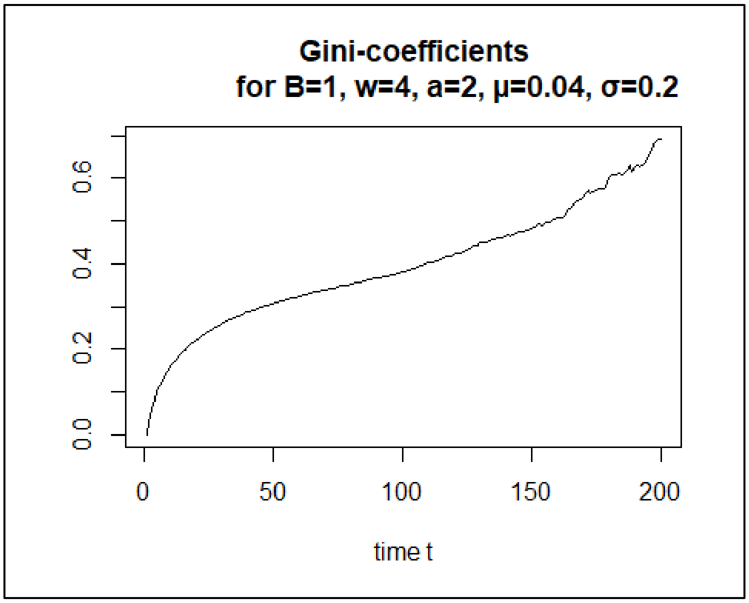
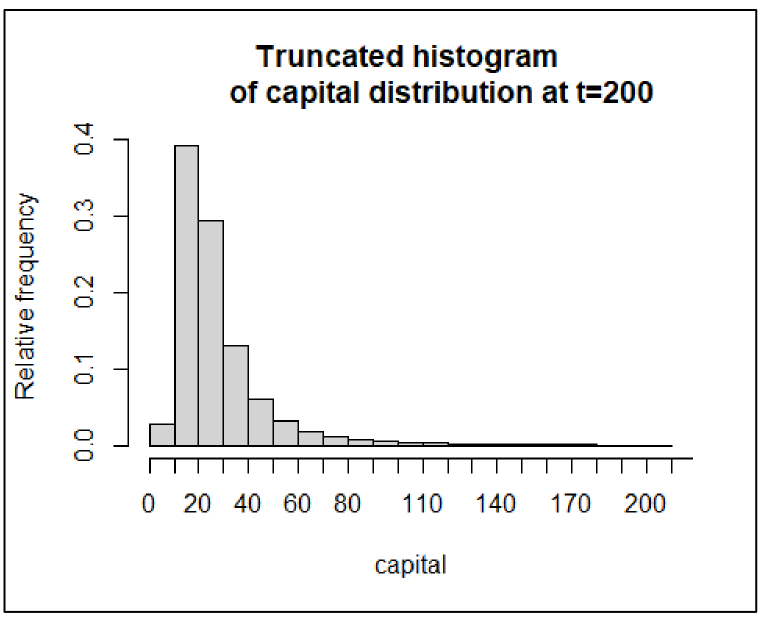
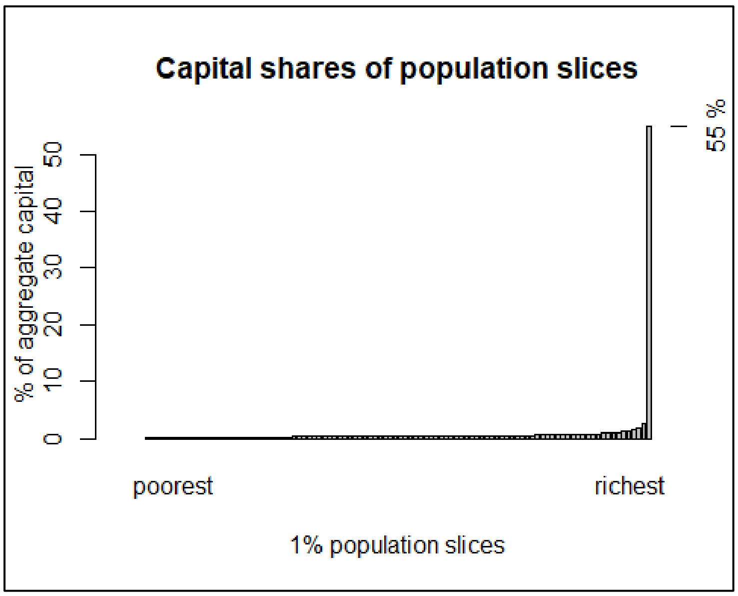


Figure 3



Aggregate capital after 200 years is $C(200) \coloneqq \sum_{i=1}^{n} b_{i,200} = 6.2 \times 10^6$, whereof almost 55% is owned by the richest 1%. The resulting Gini coefficient is $G(200)=0.69$. As the richest dynasty owns a capital of more than 650,000 at t=200, Figure 3 does not show the very long and thin right-hand tail of the histogram, and capital concentration is more clearly visualized by Figure 4, which shows the shares in aggregate capital held by all 1% slices of the population, ordered from poorest to richest, in t=200.

Figure 4



The Figures 5 to 7 show the associated time paths of aggregate capital, aggregate income $Y(t) \coloneqq \sum_{i=1}^{n}(w + r_{i,t} \cdot b_{i,t-1})$, and the capital-income ratio.

As the figures show, income and the capital-income ratio begin to fluctuate wildly with increasing concentration of capital. When capital is concentrated in the hands of a few, positive and negative returns on capital are less likely to average out than initially, when capital is distributed more evenly. One can therefore expect that the fluctuations could be reduced by increasing the number n of dynasties in the simulation.

However, quite in line with Piketty's 'fundamental force', the growth rate of aggregate income, defined as $g(t) := \frac{Y(t)-Y(t-1)}{Y(t-1)}$, is mostly below the average rate of return on capital, as can be seen from Figure 8, which shows 9-year moving averages of growth rates along with the average rates of return $\overline{r(t)} := \frac{1}{n}\sum_i r_{it}$ which, of course, fluctuate only very mildly around $\mu=0.04$.

Figure 5

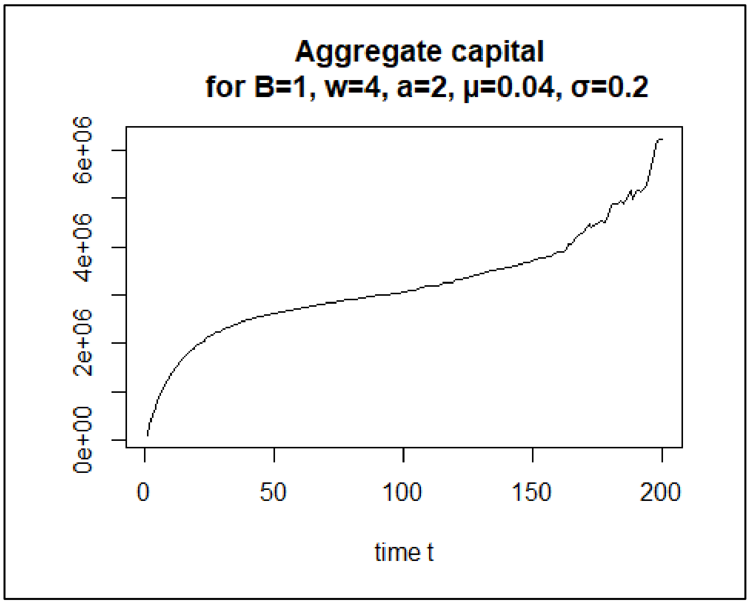


Figure 6

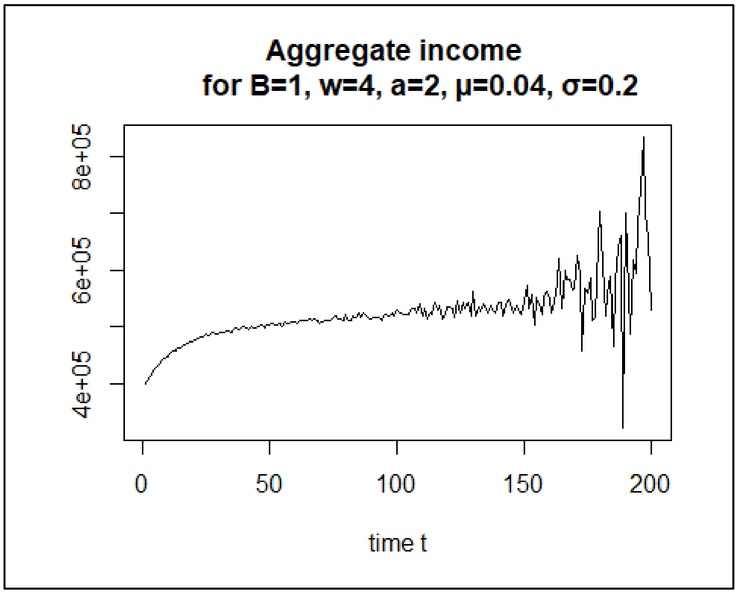


Figure 7

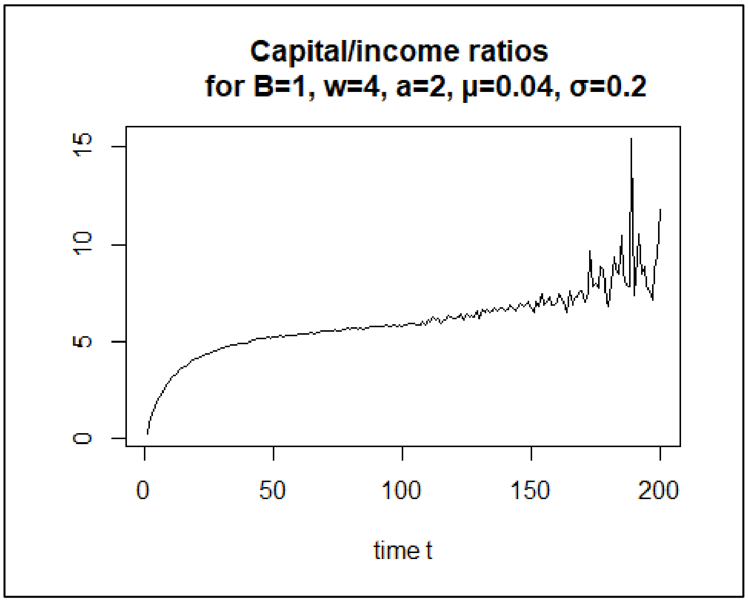
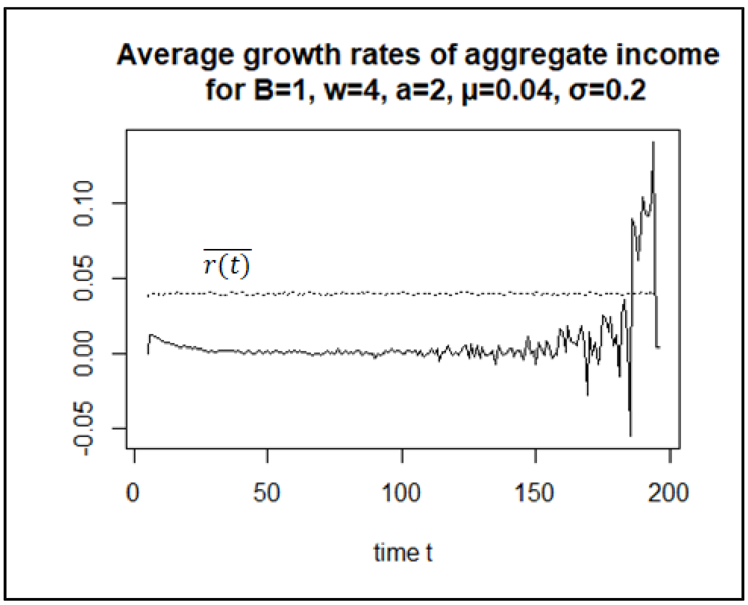


Figure 8



We can now vary the parameters of our baseline case one by one to see how this affects accumulation of aggregate capital and capital concentration.

4 Consequences of Parameter Variations

While there is some variation in the time paths of variables due to the random element in our simulations, it turns out that, compared to the baseline case, an increase in any of the parameters B, w, a, μ or σ tends to increase both accumulation as measured by the aggregate capital at t=200 and the concentration of capital as measured by either the Gini coefficient or the share of the top 1% at t=200.

E.g., if each dynasty starts with a capital endowment of B=10, but other parameters are as in the baseline case, we get the Gini time-path of Figure 9.

Now, aggregate capital is $C(200) = \sum_{i=1}^{n} b_{i,200} = 6.7 \times 10^6$, the Gini coefficient is $G(200)$=0.71, and the share of capital owned by the top 1% at t=200 is 0.58.

If, instead of initial capital B, we increase the wage (or labour productivity) to w=8, we get an aggregate capital of C(200)=644x10^6, the share of capital owned by the top 1% is 80%, and the Gini coefficient at t=200 is 0.96. Figure 10 illustrates.

Doubling the parameter *a* from the baseline value to *a*=4, we get a time path of Gini coefficients very similar to Figure 10. In this case, aggregate capital after 200 years would be 2,031 million, the Gini coefficient 0.95, and the share of the richest 1% would be 69%.

Figure 9

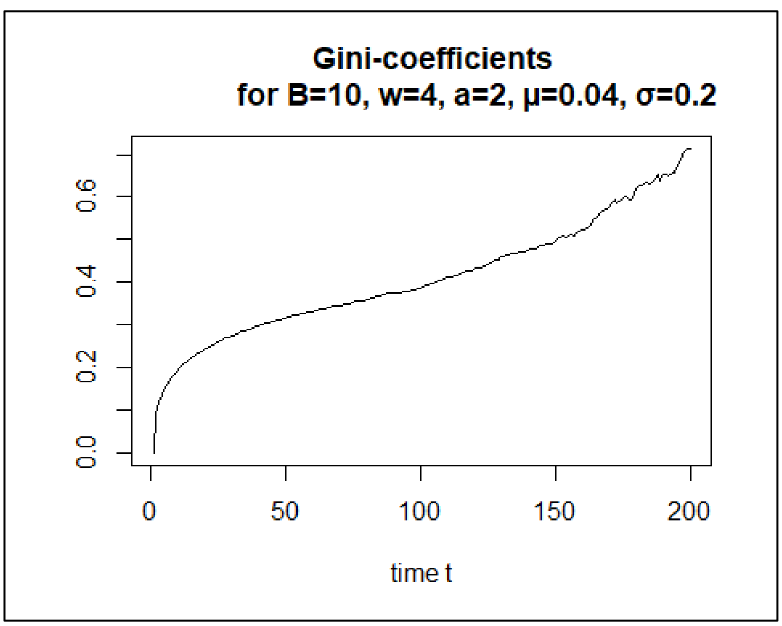
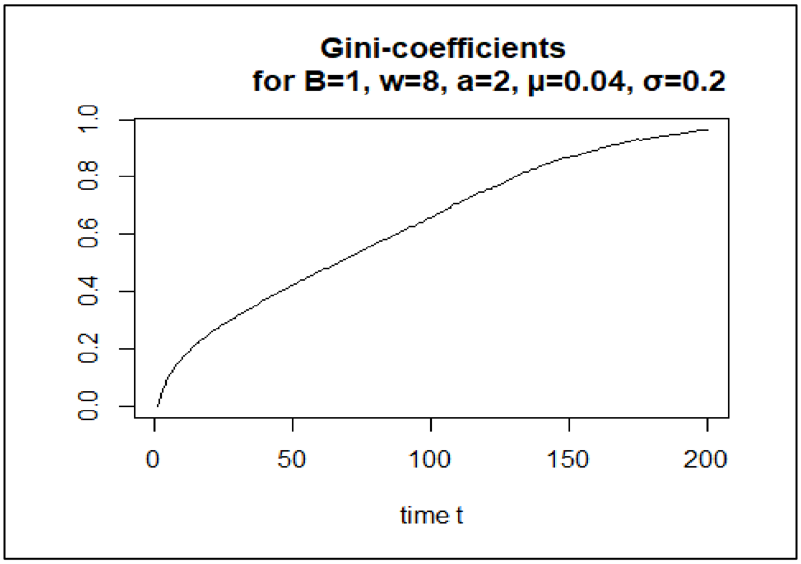
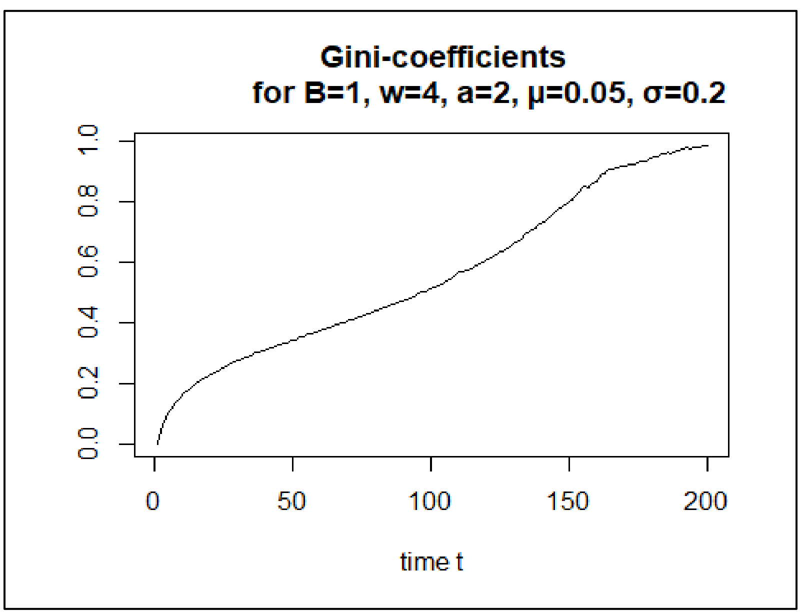


Figure 10



For rates of return on capital slightly higher than in the baseline case we get the following:

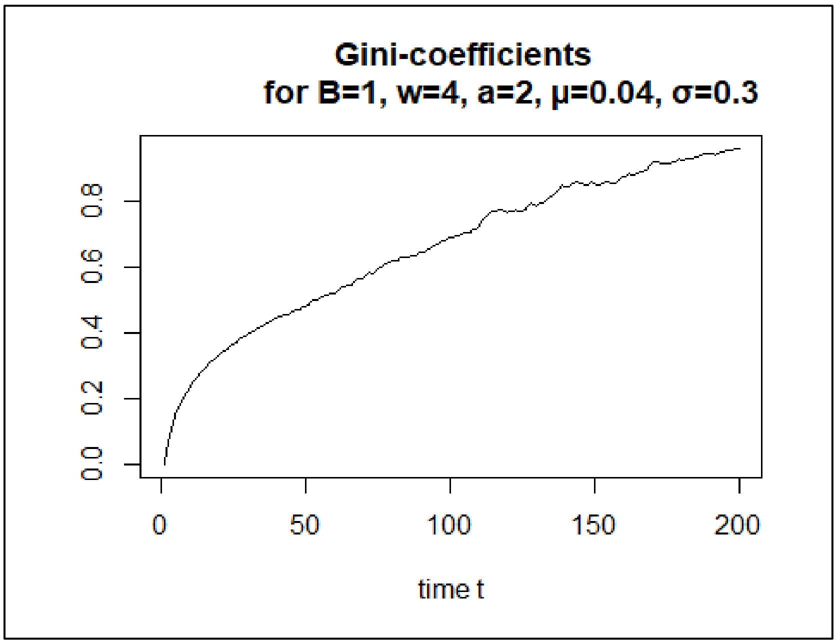
Figure 11



Now, with μ=0.05, aggregate capital is $C(200) = 189 \times 10^6$, the share of capital owned by the top 1% is 98%, and the Gini coefficient is $G(200)$=0.99.

Finally, for a higher variance in the rates of return, e.g., σ=0.3, we get Figure 12.

Figure 12



Now, aggregate capital is C(200)=43x10^6, the share of capital owned by the top 1% is 93%, and the Gini coefficient is G(200)=0.96.

Table 1 sums up the comparisons.

Table 1

	C(200)	**G(200)**	**Share of top 1%**
B=1, w=4, a=2, μ=0.04, σ=0.2	6.2 million	0.69	55 %
B=10, w=4, a=2, μ=0.04, σ=0.2	6.7 million	0.71	58 %
B=1, **w=8**, a=2, μ=0.04, σ=0.2	644 million	0.96	80 %

B=1,w=4, **a=4**, μ=0.04, σ=0.2	2,031 million	0.95	69 %
B=1, w=4, a=2, **μ=0.05**, σ=0.2	189 million	0.99	98 %
B=1, w=4, a=2, μ=0.04, **σ=0.3**	43 million	0.96	93 %

For the parameter combinations above, the time paths of other variables than the Gini coefficient are somewhat similar to those of the baseline case. In particular, for the greater part of the time span considered, ‘r>g’ holds, but not for the later part, where extreme volatility sets in. E.g., for the case of μ=0.05, we get Figure 13.

Can our model generate time paths without or with only moderate capital concentration? Certainly this is possible by sacrificing accumulation. E.g., a reduction of average rates of return to μ=0.01 would result in the time path of Figure 14.

Figure 13

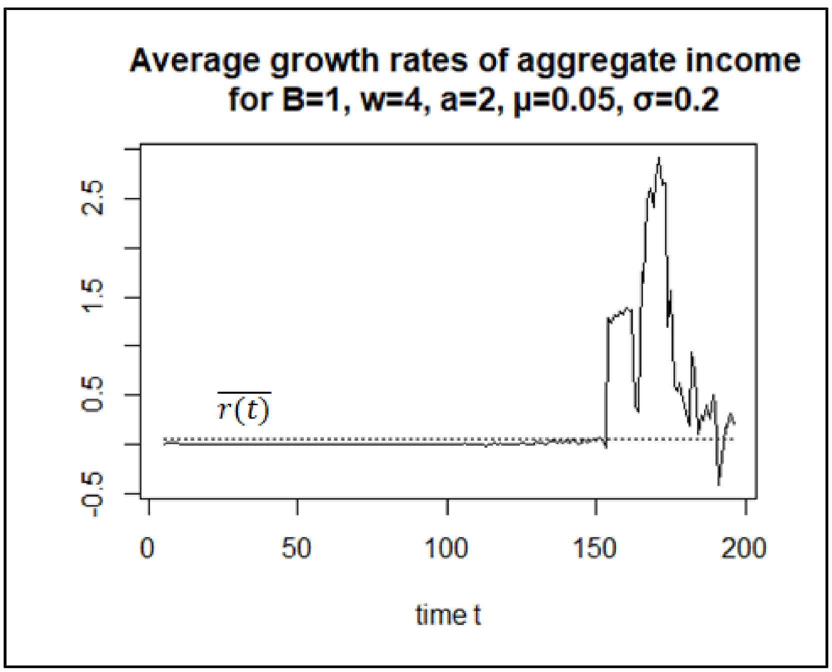
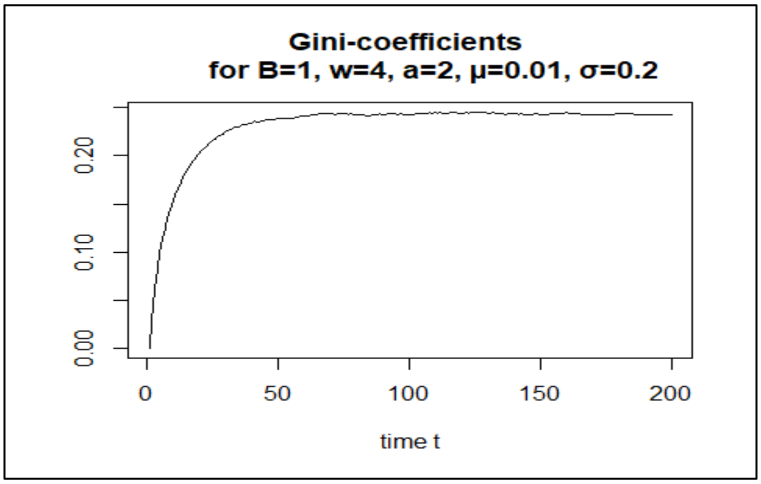


Figure 14



While in this case the Gini coefficient does not rise above 0.25, and at t=200 the capital share of the richest 1% is just 4%, aggregate capital will not grow beyond 1.9 million, individual dynasty capital ranging between 3.9 and 447. Thus, we have an essentially stagnating economy with very moderate concentration of capital. However, one also finds that growth rates of aggregate income fluctuate around zero and stay below the average rate of return quite consistently, as illustrated by Figure 15.

Thus, we have a case here where, contrary to Piketty's claim, 'r>g' over a long period of time does not 'automatically generate arbitrary and unsustainable inequalities'.

Figure 15

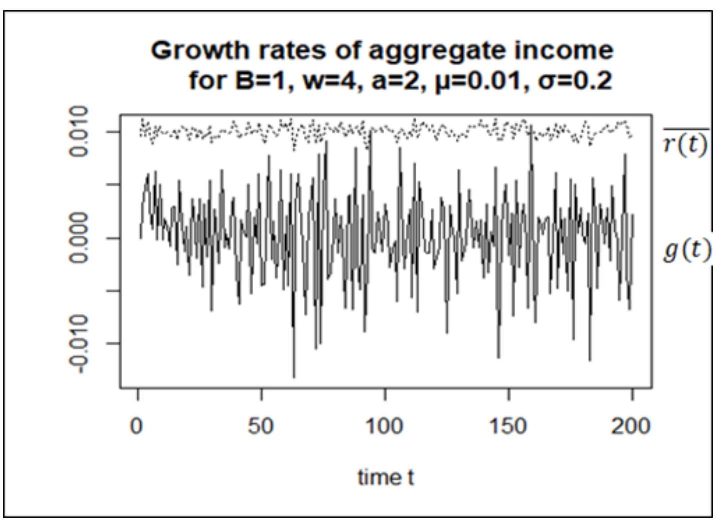
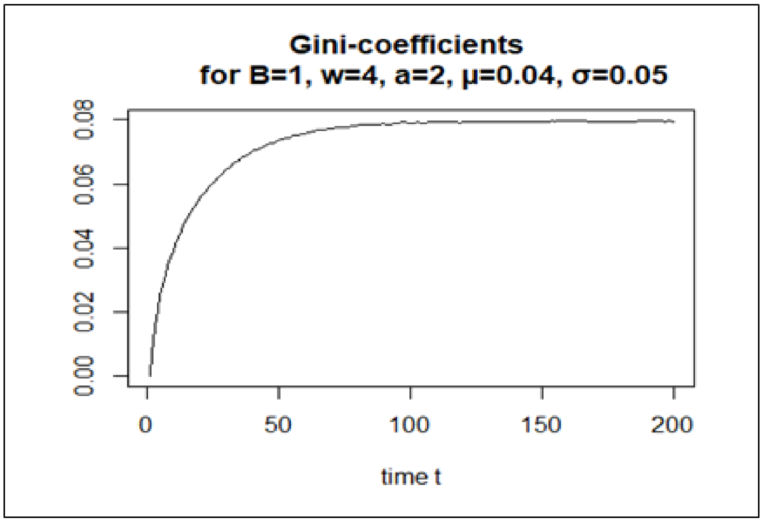
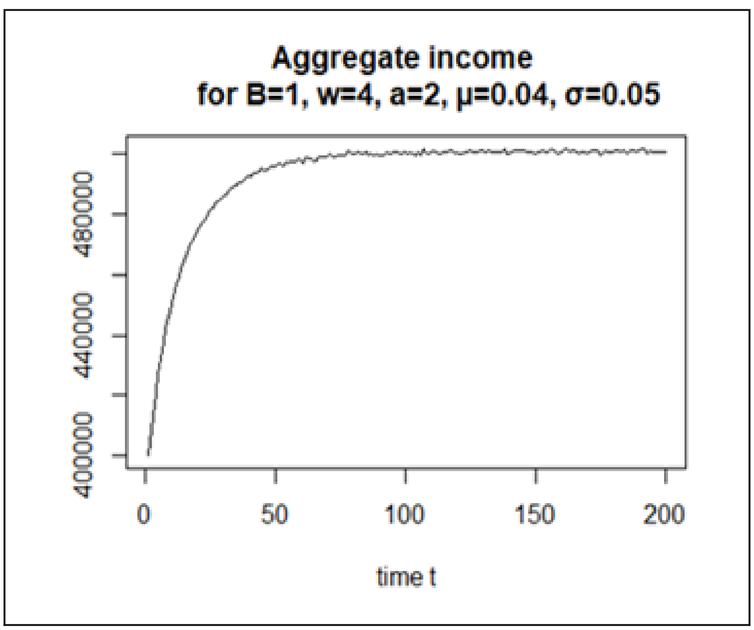


Figure 16



Not surprisingly, we also get a rather equitable capital distribution if we reduce the variance of rates of return. E.g., modifying the baseline case by assuming σ=0.05 results in the Gini coefficients of Figure 16.

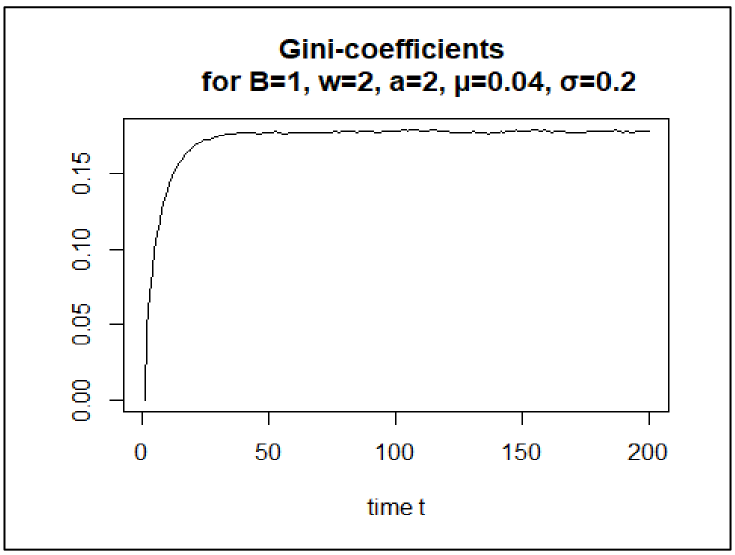
Figure 17



Again, however, the low inequality comes at a price: At t=200, average capital is only 25, and no dynasty has more than a capital of 53. And again, the rate of growth of aggregate income will be consistently below μ, the average rate of return on capital. In fact, from around t=100 onwards there will be no systematic growth of aggregate income at all, as illustrated in Figure 17 above.

It may not be quite so obvious that a labour productivity that is too low may also prevent both accumulation and concentration of capital. Assuming w=2, the Gini coefficients develop according to Figure 18, and after 200 years even the richest dynasty has a capital of no more than 52.

Figure 18

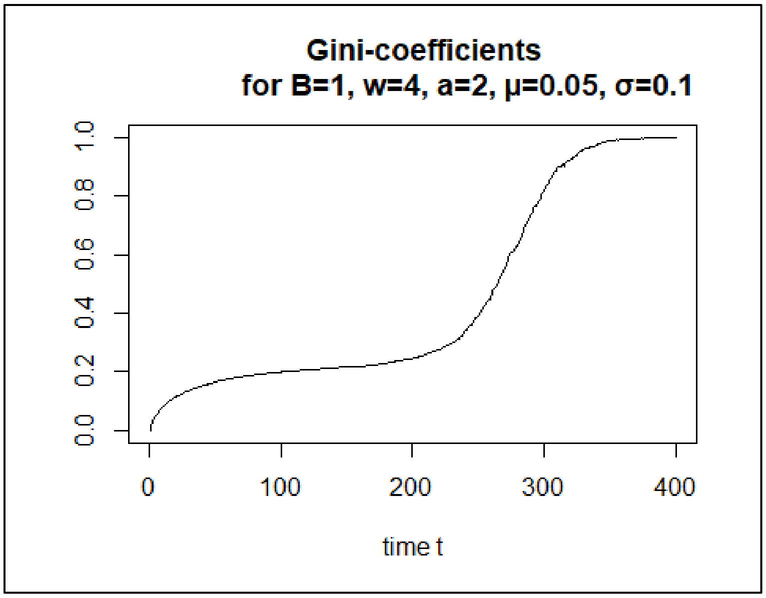


Quite a similar picture results if the parameter *a* is small. Again, these are cases where the rate of return on capital is consistently above the growth rate of income, but the concentration of capital remains negligible.

Upon reflection, these results are what one should expect: If labour productivity is 'too low' – relative to the consumption needs expressed by the consumption function - only very little capital can, on occasion, be accumulated.

Finally, there are also parameter combinations where rapidly growing capital concentration does set in, but only later than at t=200, as exemplified by the case of Figure 19.

Figure 19



5 Alternative Assumptions on Consumption Behaviour

As all of our simulations above have relied on the somewhat special type of utility function (2), they may seem to be as special as the preferences assumed. However, processes of capital concentration similar to the ones above can be obtained with many other behavioural assumptions.

The only crucial assumption about preferences seems to be that they make consumption a concave function of wealth. It is, of course, wealth rather than income that constrains someone's ability to consume, and as current wealth is empirically highly correlated with lifetime resources (e.g., Black et al. 2023), our assumption is also in line with the idea behind Friedman's (1957) permanent income hypothesis.

Concavity of the consumption function is implied by many different utility functions. E.g., if we replace the utility function (2) by

$$u(c_{i,t}, b_{i,t}) = b_{i,t} - (1 + b_{i,t} + c_{i,t}) \cdot e^{-c_{i,t}} \tag{7}$$

we get the following consumption function:

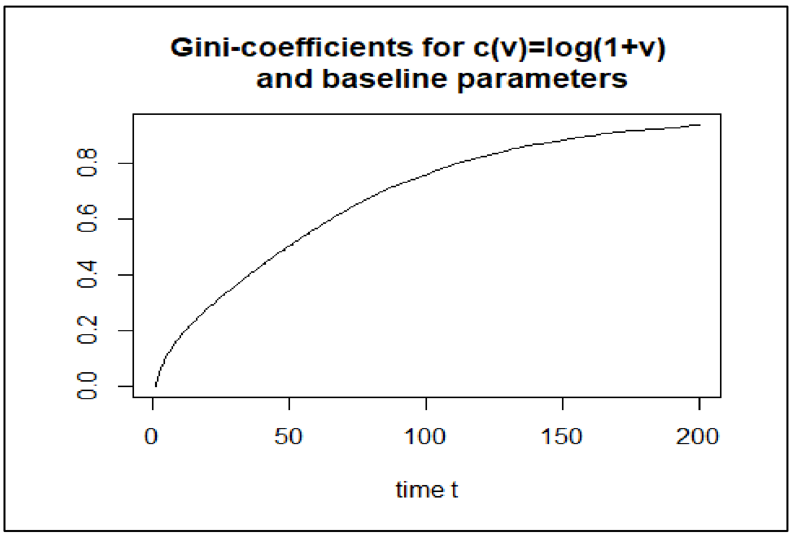
$$c_{i,t} = \log(1 + v_{i,t}). \tag{8}$$

Combining this with equation (1) and $b_{i,t} = v_{i,t} - c_{i.t}$, we get the difference equation

$$b_{i,t} = w + (1 + r_{i,t})b_{i,t-1} - \log(1 + w + (1 + r_{i,t})b_{i,t-1}) \quad (9)$$

If we modify our initial model by replacing (6) with (9), but otherwise use the parameters of our baseline case, we get the time path for Gini coefficients of Figure 20.

Figure 20



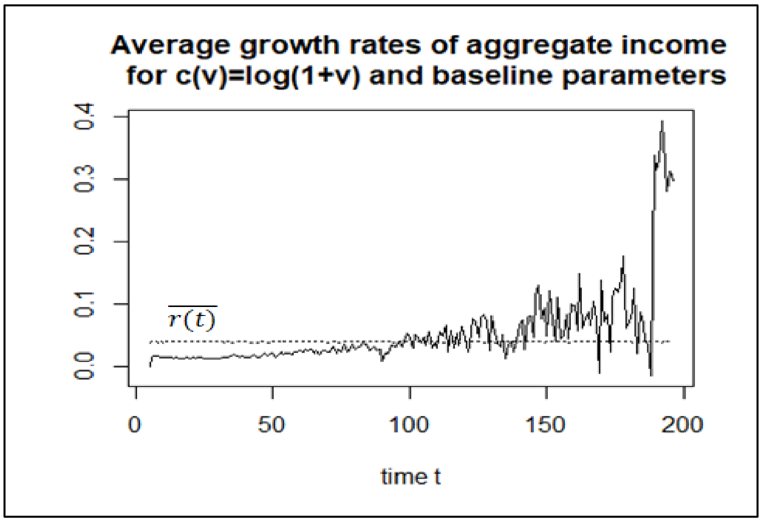
After 200 years, aggregate capital is 3.1×10^9, whereof the richest dynasty has a capital of 71×10^6, while the poorest has a capital of 8. The share of capital owned by the top 1% is 0.62 and the Gini-coefficient is 0.94. As can be seen from Figure 21, growth rates of aggregate income (more precisely, 9-year moving averages thereof) are consistently below the average rate of return only for the first half of the time span considered.

As a third example of a consumption function that is concave in wealth, we consider the following:

$$c_{i,t} = \min(v_{i,t},\ 2 + 0.01 v_{i,t}). \quad (10)$$

We get a concentration path very similar to the one of Figure 20. At t=200, we have a Gini coefficient of 0.89 and an aggregate capital of 2.1×10^9, whereof 51% are owned by the richest 1% of dynasties. In this case, for t=200, our simulation produced a Gini coefficient of 0.89, and an aggregate capital of 2.1×10^9, whereof 49% was owned by the wealthiest 1% of dynasties.

Figure 21

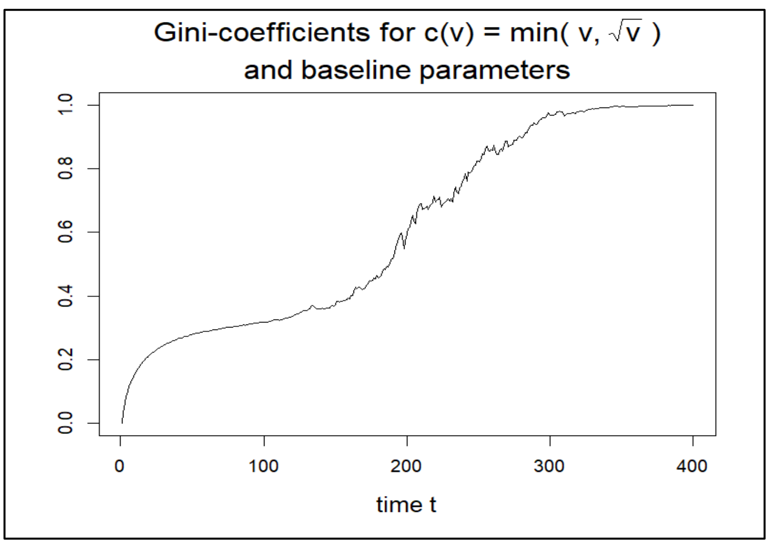


As a final example of a concave consumption function, we consider

$$c_{i,t} = \min\left(v_{i,t},\ \sqrt{v_{i,t}}\ \right). \tag{11}$$

As in the case of Figure 19, it now takes more than 200 years before the full extent of capital concentration sets in. Figure 22 illustrates.

Figure 22



It may be worth noting that the picture changes completely if we assume that consumption is not a function of wealth, but in each period a constant fraction of income, no matter how wealthy the consumer might be. As income may be negative, but consumption cannot, this assumption implies the following:

$$c_{i,t} = \max\left(0, \gamma \cdot \left(w + r_{i,t} b_{i,t-1}\right)\right) \tag{12}$$

Assuming γ =0.8, but otherwise keeping the parameter values of our baseline case, capital concentration will become stationary at a very moderate value, but so does aggregate capital, as illustrated by Figures 23 and 24.

While this is grossly at odds with the facts, no matter how stylized, it provides yet another case where 'r>g' over a long period of time does not 'automatically' result in increasing inequality. Figure 25 illustrates.

Figure 23

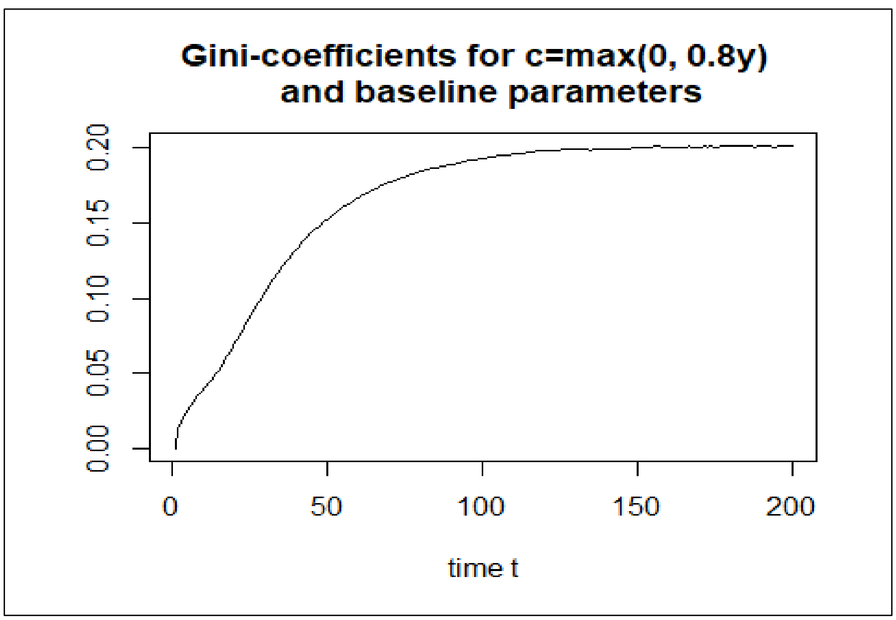


Figure 24

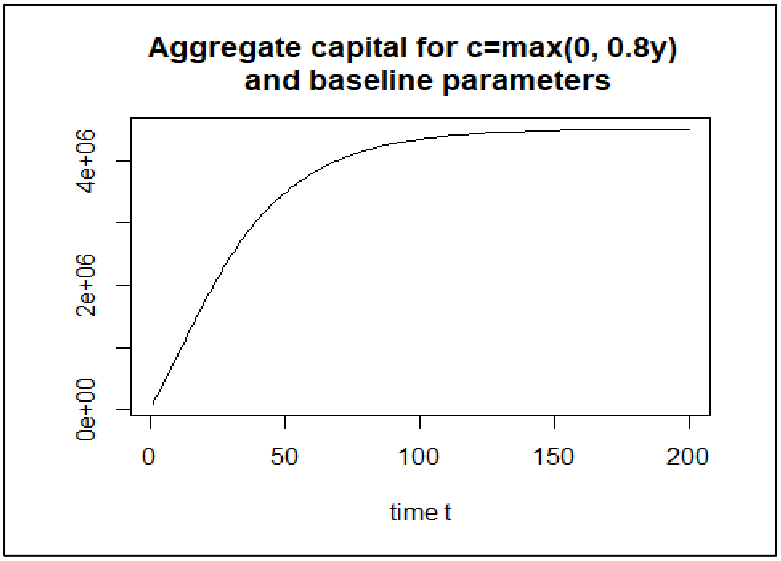
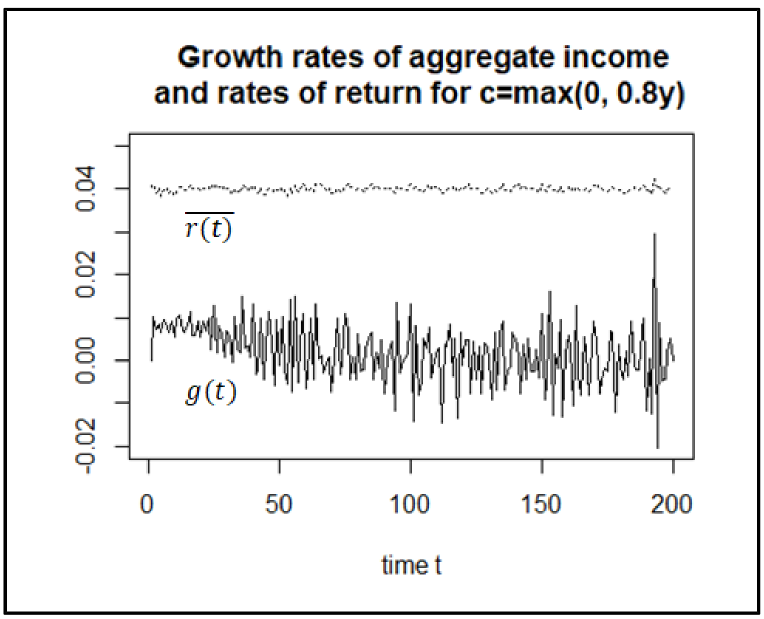


Figure 25



6 Technical Progress

It goes without saying that our model ignores many features of capitalist economies that contribute to and modify the accumulation and concentration of capital. In particular, there is no exchange between dynasties – and thus no price mechanism. Each dynasty produces for itself, as it were. Both the 'wage' w and the rates of return on capital can be thought of as physical productivities, which permits a straightforward way to simulate technical progress.

Technical progress that lets labour productivity grow exponentially can be easily included in our simulations. We modify our baseline case by assuming:

$$w(t) = w_0 * (1 + \lambda)^t \tag{13}$$

and run a simulation for w_0=4 and λ=0.02 first. It turns out that, compared to our baseline case, aggregate capital at t=200 is higher by a factor of almost 1000, but concentration of capital is higher or lower depending on the measure used: While the Gini coefficient is G(200)=0.73, the share of the top 1% is 'only' 30%.

What if the growth rate of labour productivity equals the mean rate of return on capital, i.e., if we assume $\lambda=\mu$? Not surprisingly, aggregate capital grows faster than before, but capital concentration as measured by the Gini-coefficient is still increasing quite considerably: at t=200, we have a Gini of G(200)=0.6, and the capital share of the richest 1% is 18%. Figures 26 and 27 illustrate.

Figure 26

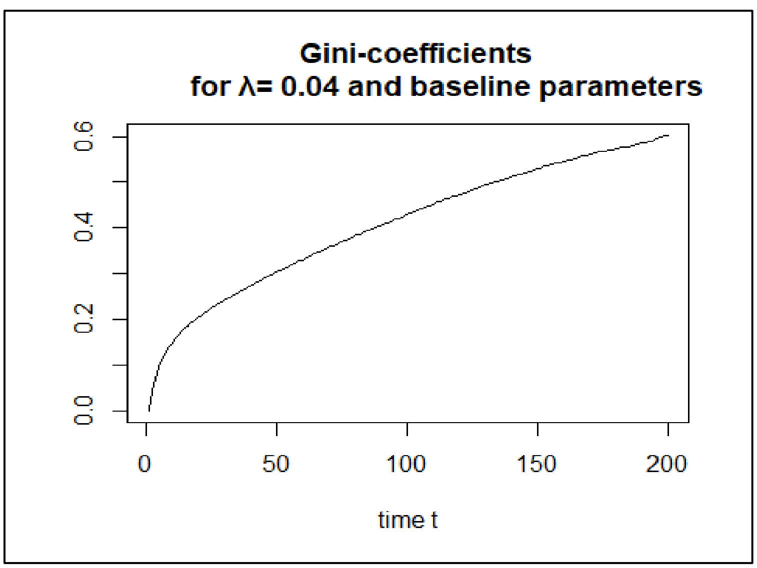
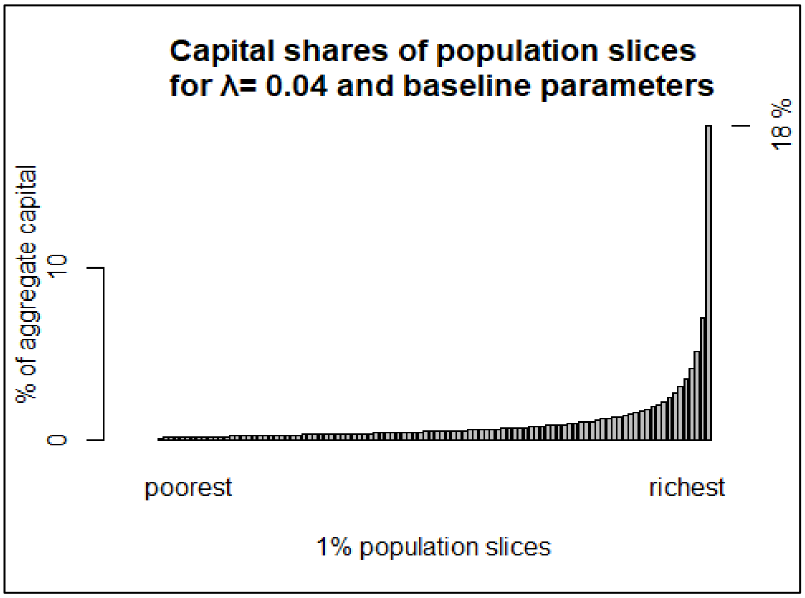
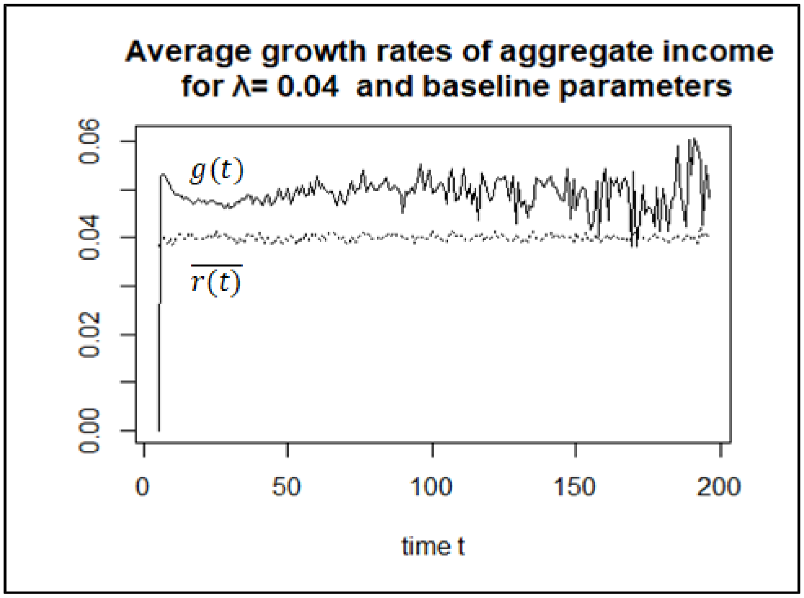


Figure 27



More remarkably, average growth rates of aggregate income are quite consistently above the average rate of return, as illustrated by Figure 28.

Figure 28



We remark that, in the present context, assumption (13) with $\lambda>0$ may also be thought of as simulating that 'the average wage is increasing if capital increases', as in the model of Stiglitz (2016).

To the extent that the return on capital reflects capital's productivity, technical progress might also lead to a gradual increase of the rate of return. The simplest way to simulate this within our model is to assume:

$$\mu(t) = \mu_0 * (1 + \delta)^t \tag{14}$$

A simulation for μ_0=0.04 and $\delta = 0.001$ shows that this would result in an acceleration, compared to our baseline case, of both accumulation and concentration of capital: For t=200, our simulation produced an aggregate capital of C(200)=20x10^6, a Gini coefficient of G(200)=0.89, and an aggregate capital share of 82% owned by the top 1%.

7 Decreasing Rates of Return

Of course, empirically observed (or estimated) rates of return on capital as discussed by Piketty are by no means just physical productivities, nor do they seem to follow a clear upward trend. Rather, it is widely held that 'the marginal productivity of capital decreases as the stock of capital increases' (Piketty, 2014, p. 215), and although it is among Stiglitz's (2016) 'new stylized facts' that returns to capital have not declined over the past 50 years or so, there have been extended periods during which the average return on capital has been decreasing. In our model, it turns out that a moderately decreasing rate of return does not change the picture very much. Using assumption (14) and μ_0=0.04, but now with $\delta = -0.002$, our simulation produced the time path of Gini coefficients shown in Figure 29.

While the random element in our simulation is more clearly visible here, and capital C(200)=3.6x10^6 is, of course, lower than in the baseline case, concentration of capital remains considerable: at t=200 the Gini-coefficient has reached a value of G(200)=0.54, and the share of the richest 1% a value of 38%. Figure 30 shows the distribution of capital shares at t=200. Reducing δ further, e.g., to $\delta = -0.003$, eventually results in a declining aggregate capital, thereby nullifying the motivation for a reduction of capital's marginal productivity.

Figure 29

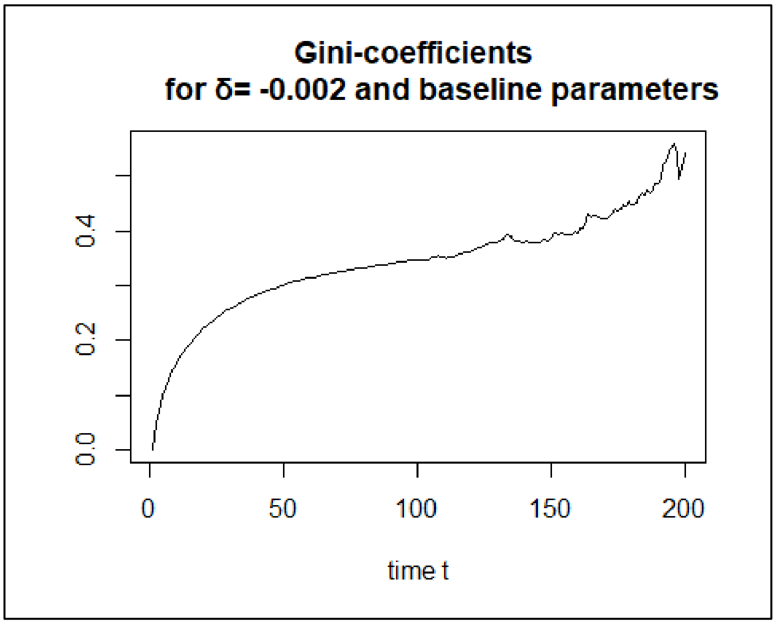
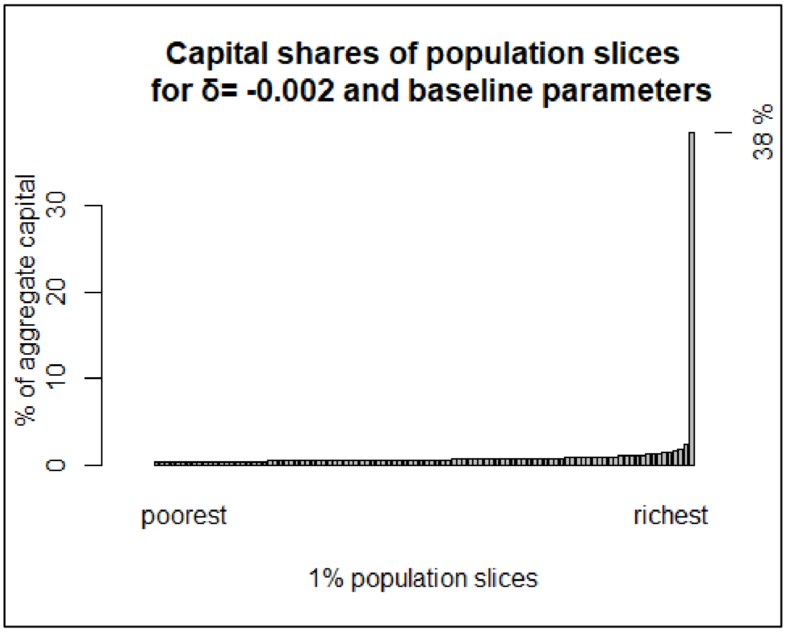


Figure 30

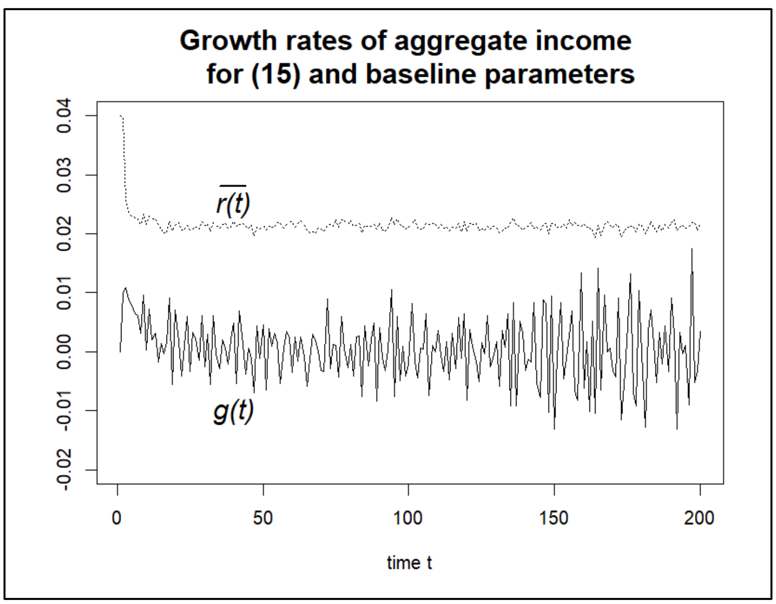


However, reducing the expected rate of return uniformly across all dynasties may not capture the intuition behind a decreasing rate of return adequately. Rather, the rate of return might depend on the individual dynasty's capital. E.g., we might assume

$$\mu(b_{i,t}) = 0.02 + \frac{0.02}{b_{i,t}} \tag{15}$$

such that initially the mean rate of return is 0.04 as in our baseline case, but declines towards 0.02 with increasing capital. In fact, this brings capital concentration to a halt at a Gini Coefficient of G(200)=0.27, but also capital accumulation at a value of C(200)=$2.1\text{x}10^6$. As can be seen from Figure 31, we have again a case where the growth rate of aggregate income consistently stays below the average rate of return – contrary to Piketty's claim that 'r>g' 'automatically generates arbitrary and unsustainable inequalities'.

Figure 31



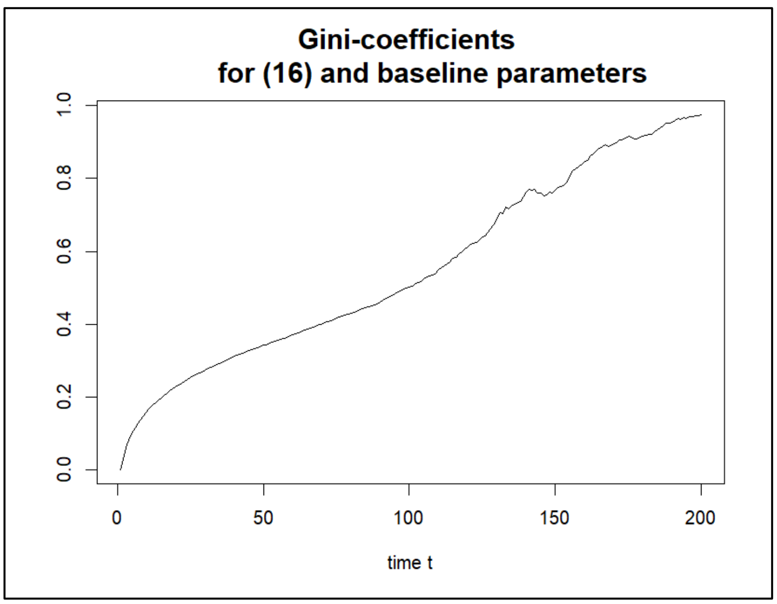
8 Positive Correlation of Wealth and Returns

Finally, there are reasons to expect an effect opposite to the one of equation (15). As Piketty (p. 430-431) points out, wealthy investors may employ consultants and financial advisors to identify better investment opportunities and thus have a higher return on investment than less wealthy investors. In fact, recent empirical findings support such a positive correlation between the wealth of an investor and the rates of return he (or she) can earn (e.g., Fagereng et al. 2020; Iacono & Palagi 2023). To simulate the effect this may have, we ran a simulation with the following assumption.

$$\mu(b_{i,t}) = 0.05 - \frac{0.01}{b_{i,t}} \tag{16}$$

Not surprisingly, drastic concentration results. At t=200, the richest 1% of dynasties own 94% of aggregate capital, and the Gini is 0.97. Figure 32 illustrates.

Figure 32



9 Conclusion

The already enormous and further increasing concentration of wealth is seen as a cause of concern by many. It is a cause of concern for mainly two reasons. The first one is the abysmal poverty at the bottom of the global wealth pyramid - poverty which could be reduced considerably by an even moderate redistribution of wealth. The second reason for concern is that the wealth of the top 1% enables the wealthy to influence public affairs to an extent that may be inconsistent with democracy. Whether or not one is worried by these concerns, the stylised fact of increasing concentration of wealth seems important enough to require more attention by economists. The insight of Fargione, Lehman and Polasky that we have elaborated on in the present paper ought to be much wider known and further explored, as it demonstrates neatly that increasing concentration of wealth may emerge even when the wealthy have no distinguishing characteristics, such as talent, that could entitle them to power and luxury when others are dying of hunger. In the model we have explored, getting rich is essentially a matter of luck. If you are lucky enough to have inherited wealth or if your investments turn out to be profitable, your chances to increase your wealth are better than if you are born into a poor family or your past investments turn out to be unlucky. These simple facts together with the assumption that as wealth increases, the share of wealth that is consumed decreases, seem to give rise to increasing concentration of wealth. Admittedly, we do not have a theorem to offer, but we conjecture that someone with the appropriate mathematical machinery may be able to provide one.

Even so, our framework neglects many issues that are of interest and of importance for an understanding of the process of wealth being handed down from one generation to the next and improving the chances of the rich to get richer. By looking in more detail at the demographics of the population in terms of age, for instance, one can find that in the short run, inheritance may well reduce rather than increase wealth concentration (Nekoei & Seim 2023). Without doubt, the role of real estate, of market and political power are as important as Stiglitz (2016) emphasizes. The part played by sheer luck, however, that our analysis highlights, seems to have been ignored all too often.

Our examples also confirm Stiglitz's argument that Piketty's 'fundamental force of divergence' does not necessarily work the way he claims. It may well happen that the average rate of return on capital is above the growth rate of income for a long time, but concentration of capital remains quite moderate. It may also happen that wealth inequality is increasing while the growth rate of aggregate income is above the average rate of return.

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