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Generalized Disappointment Aversion, Rare Disasters, and the Term Structure of Real Interest Rates

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This study models a representative agent with generalized disappointment aversion preferences in an endowment economy. This model addresses the average upward slope in U.S. real bond yields, equity premium puzzle, and equity volatility puzzle. We integrate a two-state Markov switching process for economic cycles coupled with an independent rare disaster risk. During economic expansions, disaster risk increases the probability of disappointment, thereby reinforcing precautionary saving and reducing the risk-free rate. During recessions, diminished concern about disappointment encourages borrowing and increases the risk-free rate. This pattern engenders countercyclical fluctuations in risk-free rates and accounts for the upward-sloping average yield curve.

Keywords: Generalized disappointment aversion, Yield curve, Equity premium, Equity volatility, Precautionary saving

JEL Classifications: E21, E43, G12

1 Introduction

Consumption-based asset pricing research has made significant progress in addressing several prominent puzzles, including the equity premium puzzle (Mehra and Prescott 1985), risk-free rate puzzle (Weil 1989), and equity volatility puzzle (Shiller 1981). However, the term premium puzzle (Backus, Gregory and Zin 1989) remains a challenge. U.S. historical data reveal that long-term real bonds normally yield higher returns than short-term real bonds, resulting in an upward-sloping real yield curve on average. This empirical evidence implies that investors perceive long-term bonds as risky assets and demand a positive risk premium—a result that generally diverges from the predictions of the canonical consumption-based models. As Backus et al. (1989) demonstrate, the near–i.i.d. nature of U.S. con-

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sumption growth implies that stochastic discount factors under expected utility lack the autocorrelation required to explain the long-term bond premium.

Among modern models, the habit formation model (Campbell and Cochrane 1999) addresses the equity premium and risk-free rate by substituting habit for consumption in utility. However, in Campbell and Cochrane's original framework, the risk-free rate is constant, leading to a flat yield curve.[1] The long-run risks model (Bansal and Yaron 2004) assumes Epstein-Zin preferences with a high elasticity of intertemporal substitution, and introduces AR(1) components into the aggregate consumption process to capture empirical moments of equity. Paradoxically, their emphasis on the persistent effects of bad shocks makes long-term real bonds more attractive to risk-averse investors, leading to higher prices and lower yields, and exacerbating the term premium puzzle.

One key distinction separates the term premium puzzle from other asset pricing puzzles. In the equity premium and risk-free rate puzzles, time-separable CRRA utility correctly predicts the sign of the risk premium but fails to account for its magnitude. Thus, these puzzles relate to the size of the premia, which can be addressed by calibrating Epstein-Zin preferences. By contrast, the term premium puzzle requires an explanation of the reversal in the sign of the term premium. Resolving this discrepancy demands not parameter adjustments but instead a structural shift in the underlying modeling mechanisms.

This study aims to explain the term premium puzzle, risk-free rate puzzle, equity premium puzzle, and equity volatility puzzle. We employ nonstandard preferences, known as Generalized Disappointment Aversion (GDA) preference, which evolved from Disappointment Aversion (DA) preferences proposed by Gul (1991) and was subsequently generalized by Routledge and Zin (2010). Gul's DA preference categorizes a subset of lotteries as the "disappointment set," encompassing those lotteries that arise from outcomes falling below the certainty equivalent of a given lottery. This subset yields a utility function that displays the kink introduced by the penalty associated with disappointment. DA preferences encompass expected utility as a special case in which disappointing outcomes do not occur, or when the utility penalty is null. Routledge and Zin (2010) expand upon Gul's DA preferences in two main aspects. First, the disappointment threshold was introduced as a parameter that scales the certainty equivalent, thereby enhancing flexibility in the disappointment set region. Second, they define GDA preferences using the CRRA utility function and apply this formulation to Epstein and Zin (1989) recursive utility, as also discussed in Backus, Routledge and Zin (2004).

We consider a representative agent with GDA preferences in an endowment economy. The economy is governed by two components: a two-state Markov switching process that delineates economic cycles through expansion and recession states, and disaster risk. These two components follow a binomial distribution, implying that rare disaster risks are independent of normal economic cycles. Additionally, the state transition matrix reveals that in an expansion state, the consumption growth momentum tends

[1]On modeling the term structure of real interest rates with habit formation, see Wachter (2006).

to persist, whereas in a recession state, the risk of a long-run downturn is higher.

The core idea behind this model is that two key channels influence the stochastic discount factor: (1) Intertemporal consumption smoothing motivation, which prompts the agent to borrow during expansion states; that is, the states where the expectation of consumption growth in the next period is high, and save during recession states where the expectation of consumption growth in the next period is low, which aligns with conventional models. (2) Precautionary saving motivation,[2] that stems from disappointment aversion and is controlled by the state-dependent disappointment threshold. An expansion state is associated with a higher certainty equivalent of future consumption relative to a recession state, while simultaneously being accompanied by a heightened probability of experiencing disappointment given the independence of rare disaster risk. Thus, the precautionary saving motivation prompts the agent to increase savings during economic expansions. By contrast, in a recession state, in which the probability of being disappointed is diminished, the agent becomes more willing to embrace risk and borrow money.

If (2) dominates (1), then the influence of the precautionary saving motivation prevails. Thus, the stochastic discount factor increases with consumption growth during economic expansions and decreases during economic downturns, leading to countercyclical fluctuations in the risk-free rate. Consequently, long-term bond prices exhibit procyclical fluctuations, bestowing long-term bonds with characteristics of risky assets; thereby, term premiums are requested.

Because the upward-sloping real yield curve hinges on the procyclical fluctuation in the stochastic discount factor, few consumption-based models successfully resolve this puzzle, to our knowledge. Notably, Wachter (2006) presents a renowned model in which the agent's habit formation plays a key role in generating procyclical fluctuations in the stochastic discount factor. By contrast, Suzuki (2016) introduces an economy with heterogeneous belief agents, allowing aggregate stochastic discount factors to account for real bond prices. Both models effectively address the term premium puzzle, equity premium puzzle and equity volatility puzzle. We establish that GDA preferences offer an alternative explanation for these puzzles. With appropriate parameter choices, notably, the disappointment threshold parameter, our model can effectively account for term premiums in long-term real bonds. Furthermore, by comparing GDA preferences with DA preferences and Epstein-Zin preferences across different disaster specifications, this study highlight the critical role of the combination of independent disaster risks and GDA preferences in addressing the term premium puzzle.

Earlier studies reveal that the effective risk aversion of GDA preferences contributes to substantial risk premiums in Mehra-Prescott economy, as Routledge and Zin (2010) demonstrate. Garcia and Bonomo (1993) provide a more detailed discussion of the Markov switching process for DA preferences to match the moments of equity premium and risk-free rates. The combination of long-run risk models with GDA preferences has gained popularity. Bonomo, Garcia, Meddahi and Tédongap (2011)

[2] We use the term "precautionary saving" to refer to all saving behavior aimed at mitigating future risks. This is in comparison to saving driven by the intertemporal consumption smoothing effect.

examine GDA preferences with the long-run risk model akin to Bansal and Yaron (2004). In their study, the application of GDA preferences successfully reproduced the equity moments and predictive patterns observed in the data, with the elasticity of intertemporal substitution appearing inconsequential. Delikouras (2018) focuses on modeling the standard deviation of consumption growth process, rather than the variance, as an AR(1) process. His study addresses the credit spread puzzle evident in the corporate bond market. Additionally, Augustin and Tédongap (2021) assume a constant mean and GARCH variance dynamics in the consumption growth process with GDA preferences. Their model successfully generates an average upward-sloping term structure for nominal interest rates, but the real bond yield curve retained its downward slope.

Moreover, Babiak (2024) studies a two-state Markov switching economy with the agent's learning under the assumption of GDA preferences. His study focuses on the variance and skewness of risk premium in equity returns as well as the implied volatility skewness of index options. Schreindorfer (2020) models mixture shocks in consumption and dividend processes, demonstrating that GDA preferences respond to these tail risks and can match the average return of options. Delikouras (2017) examines a consumption-based DA preference model capable of explaining the cross-section of asset prices, including equities, corporate bonds, and commodity futures. He estimates the disappointment aversion parameter using the GMM method. Further, Delikouras and Kostakis (2019) show that the indicator of consumption growth falling below its certainty equivalent, derived from GDA preferences, can serve as a single pricing factor. This single factor exhibits explanatory power comparable to that of the Fama–French 5-factor model. Other relevant studies include Liu and Miao (2015), who apply GDA preferences to a production economy, and Li, Xia and Guo (2021), who introduce gamma distribution into state-switching uncertainties to explore the historical equity premium.

Among these, the disaster specification in Dolmas (2013), who examines DA preferences, is particularly relevant for our model. Dolmas proposes a three-state Markov switching process comprising the expansion, recession, and disaster states. However, an essential difference between his specification and ours is whether a disaster can occur simultaneously with an expansion or recession state. In his specification, disaster risks are considered independent of expansion/recession states; that is, the transition probability of entering a disaster state is identical across both expansion and recession states. Because Dolmas (2013) specifies disasters as distinct states in the model, the representative agent faces only a single probability distribution corresponding to the current state. In other words, no disaster can occur concurrently with an expansion state. By contrast to Dolmas (2013), our study separates rare disasters from regular economic fluctuations by treating them as risks independent of state-switching. This setting implies that the agent always faces a "mixture distribution" that combines a disaster risk with the current state. Thus, the agent may experience disappointment even in an expansion state because a disaster is possible, while being less apprehensive in a recession state. This emotional divergence between states could contribute to a possible reversal of the slope of the yield curves. In an additional test, we compare the results to the disaster specification in Dolmas (2013).

The remainder of this paper is organized as follows: Section 2 introduces the model. Section 3 covers the data, estimation procedure, and estimation results. Section 4 discusses the asset pricing implications of the baseline model and model comparisons. Finally, Section 5 concludes the paper. Appendix A and Appendix B provide the calculation details for the baseline model and the additional test model.

2 Model

2.1 Economy

We consider an infinitely-lived representative agent in a discrete-time endowment economy with two assets: real bonds and equity. In each period, the agent consumes all dividends. We model the consumption process as

$$\Delta \ln C_{t+1} = g_{t+1} = \begin{cases} \mu(s_t) + \sigma\varepsilon_{t+1} & w.p.\ (1-p) \\ -h_{t+1} & w.p.\ \ p \end{cases}, \tag{1}$$

$$\varepsilon_{t+1} \sim i.i.d.N(0,1).$$

where p is the probability of an economic disaster occurring, and $h_t > 0$ is a random variable that determines the size of the disaster. The current economic state is denoted by $s_t = \{0,1\}$ and is governed by a two-state Markov chain with the transition matrix

$$Q = \begin{pmatrix} 1-q_{01} & q_{01} \\ q_{10} & 1-q_{10} \end{pmatrix},$$

where the transition probability from current state i to state j in the next period is q_{ij}. Without loss of generality, we assume that

$$\mu(s_t) = \begin{cases} \mu_0 & \text{if} \quad s_t = 0 \\ \mu_1 & \text{if} \quad s_t = 1 \end{cases}, \quad \mu_0 > \mu_1$$

such that state 0(1) indicates an economic expansion(recession).

Because disaster risk h_{t+1} is i.i.d. over time and independent of ε_{t+1}, it is not a state. The state of this economy can be considered as follows. When the Markov switching process enters an expansion state, the agent observes consumption growth in the next period as a mixed probability distribution. This distribution comprises a normal distribution $N(\mu_0, \sigma^2)$ weighted by $(1-p)$, along with a distribution of disaster risk denoted as h_{t+1} weighted by p. This setting represents the actual expansion state of the entire economy. Conversely, when the agent observes the Markov switching process transitioning into a recession, this implies a mixed probability distribution containing $N(\mu_1, \sigma^2)$ and the distribution of disaster risk h_{t+1}, weighted by $(1-p)$ and p, respectively. This signifies the actual recession state of the entire economy. Consequently, the model still has only two actual states, expansion or recession, corresponding to the current state of the Markov switching process. However, each state incorporates disaster risk into a mixture distribution, which is crucial for solving the model.

The equity dividend process in equation (2) is assumed to share the same disaster size and mean in economic fluctuations as the consumption process. We decompose its volatility into a common shock, denoted by ε_t, and an idiosyncratic shock, denoted by η_t. This specification aims to match the nearly three-fold higher volatility observed in dividend growth data compared to consumption data as well as the covariance between consumption and dividend growth rates.[3]

$$\Delta \ln D_{t+1} = d_{t+1} = \begin{cases} \mu(s_t) + \sigma_{d1}\varepsilon_{t+1} + \sigma_{d2}\eta_{t+1} & w.p.\ (1-p) \\ -h_{t+1} & w.p.\ p \end{cases}, \tag{2}$$

$$\eta_{t+1} \sim i.i.d.N(0,1), \quad \eta_{t+1} \perp \varepsilon_{t+1}.$$

2.2 Agent's Preferences

The agent has the following recursive GDA preferences, as in Routledge and Zin (2010):

$$V_t = \left[(1-\beta)C_t^{1-\frac{1}{\rho}} + \beta\left\{\mathcal{R}_t\left(V_{t+1}\right)\right\}^{1-\frac{1}{\rho}}\right]^{\frac{1}{1-\frac{1}{\rho}}}, \tag{3}$$

$$\mathcal{R}_t\left(V_{t+1}\right) = \left[\frac{E_t\left[V_{t+1}^{1-\gamma}\left(1+\theta\mathbb{1}\left\{V_{t+1} \leq \delta\mathcal{R}_t\left(V_{t+1}\right)\right\}\right)\right]}{1+\theta\delta^{1-\gamma}E_t\left[\mathbb{1}\left\{V_{t+1} \leq \delta\mathcal{R}_t\left(V_{t+1}\right)\right\}\right]}\right]^{\frac{1}{1-\gamma}}, \tag{4}$$

where β, γ, and ρ are the subjective time discount factor, relative risk aversion, and intertemporal elasticity of substitution, respectively. The parameter δ determines the disappointment threshold as $\delta\mathcal{R}$, whereas θ is the utility penalty that measures the disappointment aversion. $\mathcal{R}_t(V_{t+1})$ represents the certainty equivalent of the next-period utility V_{t+1}. Given the current state s_t, when $V_{t+1} \leq \delta\mathcal{R}_t(V_{t+1})$, the utility function kinks because the next period is considered as a disappointment. In addition, when the current state is an expansion, $\mathcal{R}_t(V_{t+1})$ is higher than when the current state is a recession.

We can derive the stochastic discount factor for GDA preferences as follows:

$$\pi_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\rho}}\left\{\frac{V_{t+1}}{\mathcal{R}_t\left(V_{t+1}\right)}\right\}^{\frac{1}{\rho}-\gamma}\left(\frac{1+\theta\mathbb{1}\left\{V_{t+1} \leq \delta\mathcal{R}_t\left(V_{t+1}\right)\right\}}{1+\theta\delta^{1-\gamma}E_t\left[\mathbb{1}\left\{V_{t+1} \leq \delta\mathcal{R}_t\left(V_{t+1}\right)\right\}\right]}\right). \tag{5}$$

2.3 Solution

As explained in Section 2.1, our model features only two states ($s_t \in \{0,1\}$). The solution is based on the groundwork laid by Marinacci and Montrucchio (2010). Augustin and Tédongap (2021), Li et

[3] As shown in Table 1.

al. (2021) provided specific fixed-point methods for GDA preferences in recursive form. We begin by conjecturing that the certainty equivalent to consumption ratio is a function of the current state

$$\frac{\mathcal{R}_t(V_{t+1})}{C_t} = m(s_t). \tag{6}$$

From equation (3), the utility-consumption ratio is also a function of the current state

$$\begin{aligned}\frac{V_t}{C_t} &= \left[(1-\beta) + \beta \{m(s_t)\}^{1-\frac{1}{\rho}}\right]^{\frac{1}{1-\frac{1}{\rho}}} \\ &= v(s_t).\end{aligned} \tag{7}$$

By substituting equations (6) and (7) into equation (4), $m(s_t)$ is determined by using the fixed point of

$$m(s_t) = \left[\frac{E_t\left[\{v(s_{t+1})\}^{1-\gamma} e^{(1-\gamma)g_{t+1}} \left(1 + \theta \mathbb{1}\left\{g_{t+1} \leq \ln \frac{\delta m(s_t)}{v(s_{t+1})}\right\}\right)\right]}{1 + \theta \delta^{1-\gamma} E_t\left[\mathbb{1}\left\{g_{t+1} \leq \ln \frac{\delta m(s_t)}{v(s_{t+1})}\right\}\right]}\right]^{\frac{1}{1-\gamma}}. \tag{8}$$

We can solve $m(s_t) = \{m(0), m(1)\}$ numerically using the fixed-point method, then we obtain $v(s_t) = \{v(0), v(1)\}$. Moreover, the stochastic discount factor for each state-switching case can be solved using

$$\pi_{t+1} = \beta e^{-\gamma g_{t+1}} \left\{\frac{v(s_{t+1})}{m(s_t)}\right\}^{\frac{1}{\rho}-\gamma} \left(\frac{1 + \theta \mathbb{1}\left\{g_{t+1} \leq \ln \frac{\delta m(s_t)}{v(s_{t+1})}\right\}}{1 + \theta \delta^{1-\gamma} E_t\left[\mathbb{1}\left\{g_{t+1} \leq \ln \frac{\delta m(s_t)}{v(s_{t+1})}\right\}\right]}\right). \tag{9}$$

2.4 Asset Pricing

Given the current state, the price of a one-period real discount bond is

$$B^{(1)}(s_t) = E_t(\pi_{t+1}). \tag{10}$$

We calculate the price of an N-period real discount bond recursively as follows:

$$B^{(N)}(s_t) = E_t\left[\pi_{t+1} B^{(N-1)}(s_{t+1})\right]. \tag{11}$$

The real bond yield to maturity, $Y^{(N)}(s_t)$ is determined by

$$B^{(N)}(s_t) = \frac{1}{[1 + Y^{(N)}(s_t)]^N}. \tag{12}$$

The average real bond yields can be calculated by weighting the stationary probabilities of each state in equation (12). It can also be simulated, as described in the latter section. In practice, we found a negligible difference between the two methods.

The equity price P_t and equity price-dividend ratio $\frac{P}{D}(s_t)$ in the current state are

$$P_t = E_t[\pi_{t+1}(D_{t+1} + P_{t+1})], \tag{13}$$

$$\frac{P}{D}(s_t) = E_t\left[\pi_{t+1} e^{d_{t+1}} \left(1 + \frac{P}{D}(s_{t+1})\right)\right]. \tag{14}$$

The details of calculations for the recursive utility, S.D.F, and asset prices are provided in Appendix A.1 and A.2.

3 Estimation

3.1 Data

Our model aims to capture the slope of the average real bond yield curve. However, it is important to acknowledge that, in estimation, real bond yields vary across different calculation methods and time periods, often due to limitations in data availability. Treasury Inflation-Protected Securities (TIPS) are widely regarded as real bonds, but data are scarce, as TIPS were not introduced until 1997, with maturities restricted to 5, 10, and 30 years initially. Additionally, high illiquidity in the early years of TIPS may have depressed prices. The TIPS market stabilized, and the data became reliable by 2003.[4]

To address this, we adopt the method employed by Suzuki (2016) to construct the real bond sample. Specifically, we construct a comprehensive dataset by integrating TIPS yields from 2003.Q1 to 2020.Q2 with estimated real bond yields in Chernov and Mueller (2012), covering the period from 1971.Q3 to 2002.Q4. Because Chernov and Mueller (2012) only provide estimation results for 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, and 10-year real bonds, our sample includes yields for maturities ranging from 1 to 10 years, and treat the 1-year real yields as risk-free rates.

The combined sample spans the period 1971.Q3 to 2020.Q2, covering U.S. recessions in the 1990s, early 2000s, and the 2008 Lehman shock. The 1-year yields experienced significant initial increases during these recession periods, followed by a rapid decline. Over the full sample period, the average risk-free rate is 1.54%, with an average 10-year yield spread above the one-year yield amounting to 0.7%. Figure 1 (a) shows the average yield curve of the sample, which exhibits an upward sloping and nearly linear pattern. In comparison, Figure 1 (b) depicts the average yield curve in the post-2003 TIPS data with maturities extending for up to 20 years. This term structure is upward sloping and, more importantly, exhibits a trend of convergence in yields for longer maturities. This feature is not captured in our sample, indicating that the overall term structure of interest rates exhibits concavity.

To calibrate the real consumption and dividend processes for the U.S. economy during our real bond sample period, we use annual data on real personal consumption expenditure and monthly data on real dividends sourced from Robert J. Shiller's website.[5] These datasets are employed to estimate the parameter values in equations (1) and (2). Shiller's consumption data are available only until 2009.Q2, so we extend the consumption growth data series to cover the period up to 2021 using information from tables 2.3.6 and 7.1 of the National Income and Product Accounts obtained from the Bureau of Economic Analysis (BEA). Our estimation of consumption and dividend processes encompasses the period 1971–2021.

We present the statistics of the real consumption and real dividend samples in Table 1. On average, the logarithmic annual growth rate of per capita consumption was approximately 1.94%, with a stan-

[4]These limitations of TIPS have been examined in studies by Gürkaynak, Sack and Wright (2010) and D'Amico, Kim and Wei (2018).

[5]http://www.econ.yale.edu/ shiller/data.htm

Figure 1. This figure displays the average yield curves in the full sample and the TIPS data.

(a) Average yield curve in the full sample (1971-2020) (b) Average yield curve in the TIPS data (2003-2020)

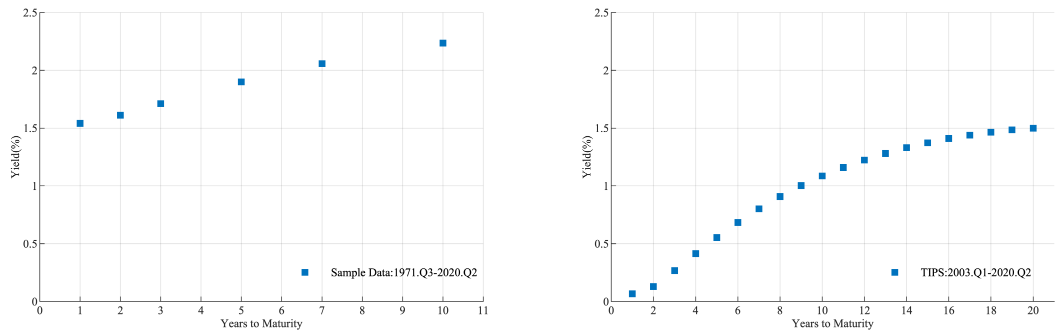


Table 1. Statistics for the log consumption growth rate and log dividend growth rate using data from Shiller's website.

	Mean	S.D.	Min	Max
$\Delta \ln C_{t+1}$ (1971-2021, annualized)	1.94%	1.98%	-3.41%	7.83%
$\Delta \ln D_{t+1}$ (1971-2021, annualized)	2.15%	6.33%	-26.24%	15.17%
$\text{Corr}(\Delta \ln C_{t+1}, \Delta \ln D_{t+1}) = 0.1223$				

dard deviation of approximately 1.98%. This contrasts with the logarithmic annual growth rate of real dividends, which averages about 2.15%, akin to the real consumption growth rate but with significantly higher volatility at 6.33%.

During the sample period, the most severe economic contraction observed is approximately -3.41%, which suggests that disastrous contractions are absent in U.S. consumption. Therefore, following Barro and Ursúa (2008) who present the empirical results of disastrous contractions in consumption growth across 22 countries, we set the mean disaster size at -20% and the probability of a disaster occurring at 3.5%. Thus, our parameter estimation is based solely on the economic Markov switching process.

3.2 Estimation Results

We apply the Markov Chain Monte Carlo method with a Gibbs sampler to the U.S. log-consumption growth rates to obtain parameter estimates of $(\mu_0, \mu_1, \sigma, q_{01}, q_{10})$. The number of iterations was 50,000, and the number of burn-in samples was 2,000.

The results of MCMC estimation are reported in Table 2. The expected annual consumption growth rate is approximately 2.5% during expansions and nearly zero during recessions. The conditional standard deviation of the consumption growth rate is slightly less than 2%. Conditional on an expansion state, there is an 81% probability that the next period will also be an expansion, signifying stable economic growth in U.S. economy. Conversely, once the economy enters a recession state, the probability of recovery in the next period is 45%, which indicates the onset of a downturn. The unconditional probability of being in the expansion state, calculated as $q_{10}/(q_{01} + q_{10})$, is approximately 70%. From Table 1, the correlation between the growth rates of consumption and dividend in our sample period is 0.1223. We therefore directly calculate the common and idiosyncratic shock components as $\sigma_{d1} = \text{Cov}(g_t, d_t)/\sigma \approx 0.01$ and $\sigma_{d2} = \sqrt{\text{Var}(d_t) - \sigma^2} \approx 0.063$.

Table 2. This table represents the MCMC estimation results of the consumption process in equation (1).

	Mean	S.D.	[97.5%	50%	2.5%]
μ_0	2.49%	0.38%	3.25%	2.49%	1.76%
μ_1	0.06%	0.77%	1.50%	0.09%	-1.56%
σ	2.02%	0.23%	2.65%	1.96%	1.75%
q_{01}	18.70%	3.74%	26.58%	18.49%	11.94%
q_{10}	45.34%	4.88%	54.88%	45.30%	35.88%

Regarding the agent's preferences, the limited number of studies simultaneously exploring disappointment aversion and threshold parameters of GDA preferences, coupled with the influence of these parameters on effective risk aversion, has led to controversy regarding parameter settings in the early literature. For instance, Routledge and Zin (2010) tested the standard model, but they required the threshold parameter δ to exceed 1 in positive autocorrelation consumption processes. Bonomo et al. (2011) set the threshold parameter δ slightly below 1 (0.989), a value very close to DA preferences. Additionally, they specified an RRA of 7.47 and a lower disappointment aversion of approximately 2.45.[6]

We chose the baseline parameters for the following reasons. If one chooses a high penalty for disappointment aversion, then choosing a smaller threshold parameter δ would at most harmlessly reduce the model to Epstein-Zin case. Conversely, a threshold parameter close to 1 may overly emphasize the effect of disappointment aversion, potentially resulting in unrealistic agent behavior. For example, as Ang, Bekaert and Liu (2005) show, under DA preferences (i.e., $\delta = 1$), it may be optimal to avoid holding risky assets if the penalty parameter is too high. Therefore, we initially keep the RRA at the level proposed by Bonomo et al. (2011), setting it to 7.4. We then set the utility penalty θ to a maxi-

[6]Bonomo et al. (2011) set their α to 0.29, which corresponds to approximately 2.45 in terms of our θ due to the differences in parameter notation.

mum value of 8.41 as a baseline. This value was estimated in Delikouras (2017) and also adopted in the model of Babiak (2024). The high penalty parameter allows for the consideration of a much lower threshold value δ, which is set to 0.8. Further comparisons involving lower RRA, lower disappointment aversion, higher threshold parameters, and DA preference model are discussed after the baseline model.

Table 3. Parameter values for the baseline model simulation

Consumption process						
μ_0	μ_1	σ	q_{01}	q_{10}	h	p
0.025	0	0.02	0.19	0.45	0.20	0.035
Dividend process						
σ_{d1}	σ_{d2}					
0.01	0.063					
Agent's preferences						
δ	θ	β	γ	ρ		
0.8	8.41	0.9412	7.4	2		

Other parameters used in baseline model simulation are summarized in Table 3. For disasters, we first assume that the disaster size is constant ($h = 0.2$). In Section 4.4, we allow h_t to be a random variable and examine its impact on asset prices. The simulation procedure follows the methodology described by Cecchetti, Pok-Sang and Nelson (1990). A simulation sample spans 50 years. We compute the mean and variance of the equity returns and risk-free rates for each sample. We repeat this procedure 20,000 times to obtain the average values of the asset return moments.

4 Results

This section presents the asset pricing results derived from our model. Notably, this model generates an upward-sloping average yield curve. Compared to alternative models, we explain the agent's saving and borrowing behavior which underlies the slope of the bond yield curve, and emphasize the role of GDA preferences and disaster specification.

4.1 Baseline Model

As Table 4 shows, the baseline model generates a risk-free rate of 1.51%, which closely approximates the historically reported 1-year real interest rate of 1.54% during the sample period. The average unconditional equity return implied by the model is 7.58% and the equity premium is 6.05%, which closely match the data. The standard deviation of the equity returns at 10.38% captures two-thirds of the observed data. The empirical data reveal an average price-dividend ratio of 42.95, accompanied by a substantial standard deviation of 18.35. In our model, although the P/D ratios are slightly procyclical, they are 16.69 in the expansion state and 16.45 in the recession state. Thus, our model does not address the issue of P/D ratios.

A potential explanation for these results is the offsetting effect of expected future dividend growth and hedging motivation. During expansion, the agent anticipates an increase in the next period's dividends, which typically leads to an increase in equity prices. However, akin to the precautionary saving motivation, disaster risk amplifies the GDA preference agent's hedging motivation, thereby exerting downward pressure on equity prices. Conversely, in a recession, decreased expectations of dividend growth reduce equity prices but hedging motivation diminishes. Hence, it is conceivable that the P/D ratios do not display notable discrepancies.

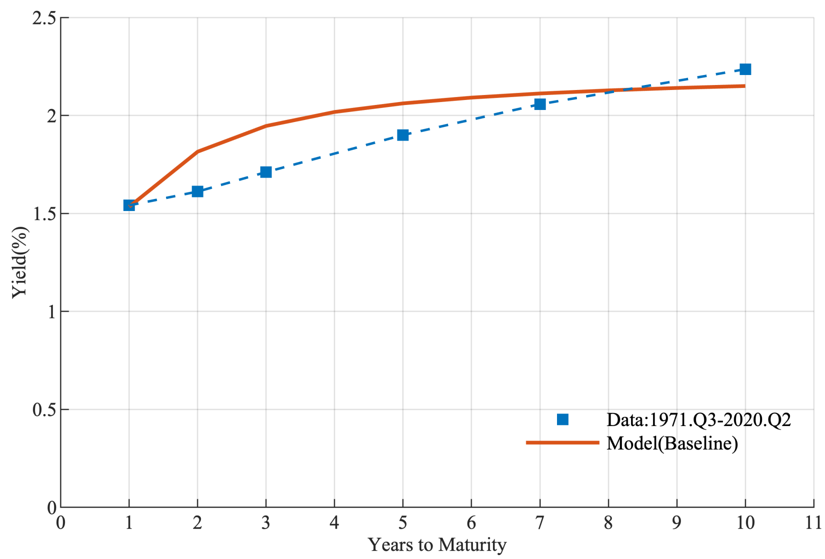
Table 4. This table presents the unconditional asset pricing results for the risk-free rate, equity return and premium, and P/D ratio in baseline model simulation with the parameters in Table 3. The results are compared with the observed data (95% confidence intervals over simulation samples in square brackets).

		Risk-free Rate	Equity Return	Equity Premium
Simulation	Mean	1.51%	7.58%	6.05%
		[0.88%,2.19%]	[6.18%,8.80%]	[4.37%,7.61%]
	S.D.	1.64%	10.38%	10.62%
		[1.20%,1.85%]	[8.02%,12.87%]	[8.29%,13.08%]
Data	Mean	1.54%	7.28%	6.29%
	S.D.	1.99%	16.51%	16.17%
		P/D		
Model Calculation		16.69 (in extension state)	16.45 (in recession state)	
Data		42.95(s.d. 18.35)		

The average yield curve implied by the baseline model is depicted in Figure 2 and the corresponding calculation results are summarized in Table 5. The baseline model effectively addresses an average positive term spread of 0.62% for 10-to-1 year bonds over the sample period. The model generates an upward-sloping yet concave term structure. By contrast, the observed yield curve exhibits a linear upward trend for maturities up to 10 years, whereas concavity emerges when maturities extend to 20 years (Figure 1).

One probable reason is that the concavity of the term structure arises from the persistence of economic states or shock dynamics in the consumption setting. Because we model consumption using a Markov switching process, when the economy enters a recession state, short-term future states can be predicted by the transition probability matrix. That is, the conditional probabilities of remaining in a recession in the second and third years differ. However, in the long-term, these probabilities gradually converge to their unconditional values. Consequently, consumption forecasts beyond 7 to 10 years in this model become nearly indistinguishable, ultimately flattening the model-implied term structure. By contrast, the linear upward trend in the observed term structure persisted for approximately twice as long (up to 10 years) before flattening. This suggests that states persist longer and influence interest rates more than 10 years ahead in the real economy.

Figure 2. This figure illustrates the average real yield curve as implied by the baseline model along with the data sample for comparison.

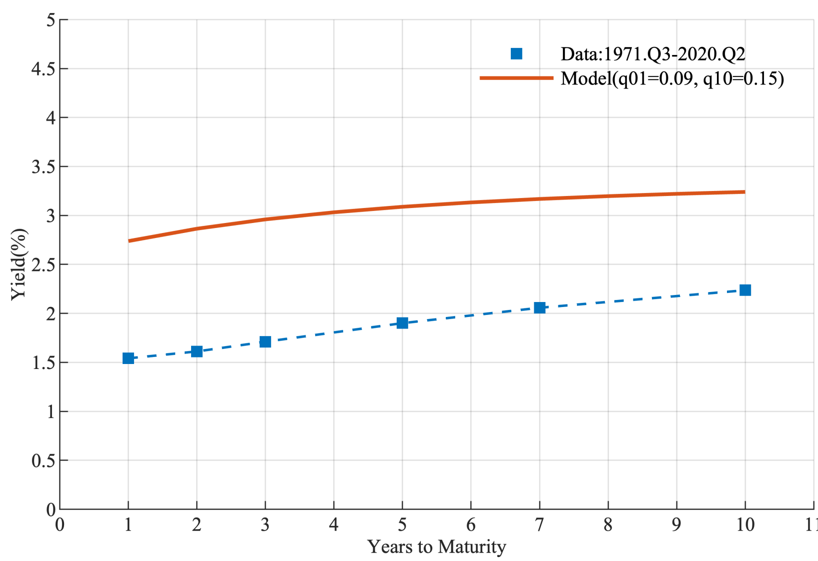


To capture the concavity of the term structure shown in the data, one can calibrate the current model using a transition probability matrix that reflects longer-persistent economic states (e.g., $(q_{01}, q_{10}) = (0.09, 0.15)$, see Figure 3). However, such matrix is not consistent with the MCMC estimation results obtained from the consumption data. An alternative approach involves considering shock dynamics; for example, the long-run risk model in Bansal and Yaron (2004) that introduces persistent shocks may be capable of addressing this issue.

Table 5. This table reports the unconditional real interest rates for 1-, 2-, 3-, 5-, 7-, and 10-year bonds calculated using the baseline model with the parameters from Table 3. The results are compared with observed data.

Interest Rate	1-Year (Risk-free)	2-Year	3-Year	5-Year	7-Year	10-Year
Model Calculation	1.53%	1.82%	1.95%	2.06%	2.11%	2.15%
Data	1.54%	1.61%	1.71%	1.90%	2.05%	2.23%
(s.d.)	(1.99%)	(1.8%)	(1.69%)	(1.52%)	(1.40%)	(1.26%)

Figure 3. This figure illustrates the average real yield curve in the model when set $(q_{01}, q_{10}) = (0.09, 0.15)$. All other parameters are consistent with the baseline model.



4.2 Agent Behavior Behind Upward-Sloping Yield Curves

We present a yield curve decomposition for both expansion and recession states by comparing the model results across Epstein-Zin, GDA, and DA preferences. This decomposition provides insights into the agent's saving and borrowing behaviors, and helps elucidate the underlying reasons behind the observed yield slopes. Furthermore, we construct the 3-state Markov-switching model test to compare our disaster risk specification with that of Dolmas (2013), emphasizing the key distinction between modeling disasters as independent risks versus independent states and analyzing their differing implications.

In the absence of disaster risk and with the agent following Epstein-Zin preferences, as per the conventional model, the intertemporal consumption smoothing motivation dominates the precautionary saving motivation. In the expansion state, the agent's marginal utility will decrease in the future, prompting an inclination towards higher current consumption and borrowing. This increases the risk-free rate. Conversely, in the recession state, the agent's propensity to save in preparation for possible future consumption reduction leads to a decline in the risk-free rate. This behavior is illustrated in Figure 4 (a).

The introduction of disaster risk strengthens the agent's precautionary saving motivation and affects the risk-free rate, as illustrated in Figure 4 (b). In response to disaster risks, the agent's borrowing during an expansion state is curtailed, whereas savings in a recession state increase, leading to a decline in all yields. However, provided that the agent adheres to Epstein-Zin preferences, the impact of such precautionary saving motivation is not substantial enough to override the dominance of consumption smoothing motivation. Consequently, the real yield curve exhibits a downward slope.

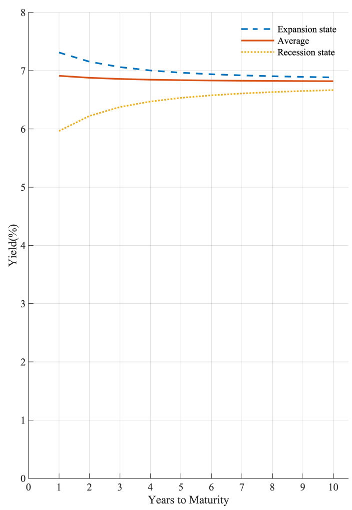
Shifting our focus to GDA preferences, when the economy lacks disaster risk, the choice of threshold parameter δ at 0.8 suggests that disappointment aversion is of minimal concern. This setting aligns GDA preferences with the Epstein-Zin framework, as Figure 4 (c) illustrates.

We want to highlight the significance of Figure 4 (d), where disaster risks enter the economy and the agent operates under GDA preferences. In this case, the motivation for precautionary saving driven by disappointment aversion supersedes the dominance of consumption smoothing motivation. This inversion of the motivational hierarchy is attributed to the higher certainty equivalent in an expansion state relative to a recession state, coupled with the independence of disaster risk from economic fluctuations. In the expansion state, disastrous outcomes falling below the disappointment threshold intensify the possibility of future disappointment. This triggers a substantial upsurge in marginal utility and the expected stochastic discount factor, compelling the agent to prioritize savings despite the projected growth in consumption. Conversely, in the recession state, given that disastrous outcomes do not fall below the disappointment threshold, the increase in marginal utility prompted by a future recession is less severe than that prompted by the expectation of disappointment in the expansion. Diminished apprehension about potential disappointment renders the agent more amenable to borrowing during a recession. Consequently, the dominance of the precautionary saving motivation leads stochastic discount factors in expansion states to exceed those in recessions. This results in countercyclical risk-free rates and procyclical bond prices. This dynamic renders long-term bonds risky, and necessitates a positive term spread.

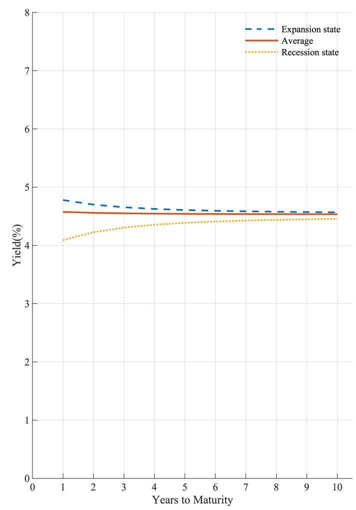
Testing how GDA preferences perform under the disaster specification in Dolmas (2013) (a Rietz-like model) clarifies the features of our disaster model. The model in Dolmas (2013) treats disasters as Markov states with transition probability $(0.5, 0.5, 0)$ from disaster to expansion, recession, disaster states, and identical probability p from expansion/ recession to disaster states (hence, "independent

Figure 4. This figure decomposes the yield curves in each state under different disaster and preference models. Panels (a)–(d) are based on our economic model (Eq.(1)) with the disaster probability set to 0 in (a) and (c) and to 0.035 in (b) and (d). Panel (e) tests a 3-state economic model using the Dolmas's specification (model details in Appendix B) with all parameters consistent with the baseline model (Panel (d)).

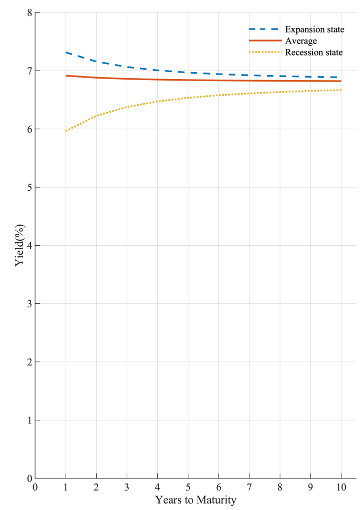
(a) Non-disaster model with E-Z preferences



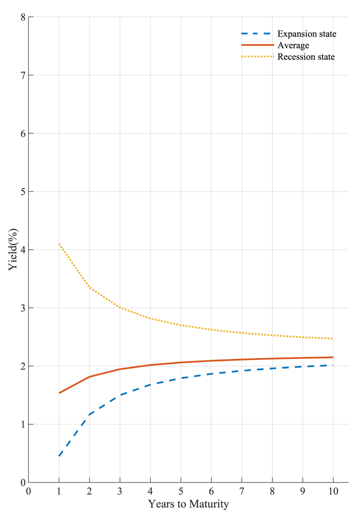
(b) Disaster model with E-Z preferences



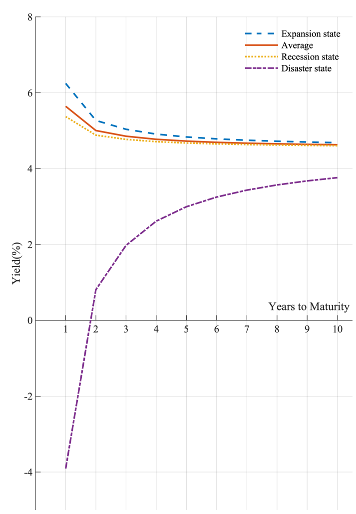
(c) Non-disaster model with GDA preferences



(d) Disaster model with GDA preferences



(e) 3-state model with GDA preferences



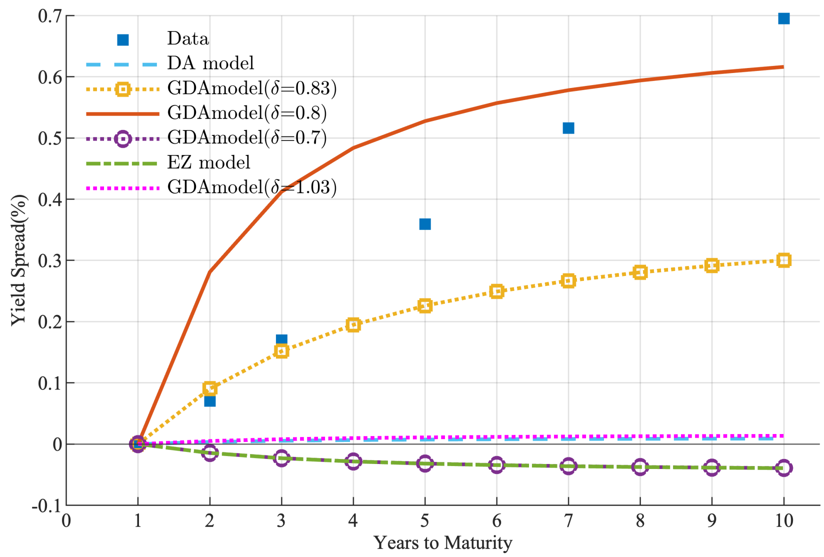
disaster state"). We thus construct a 3-state model (details in Appendix B) and test it with recursive GDA preferences.[7] All parameters are consistent with the baseline calibration in Table3. Figure 4 (e) shows the results from the 3-state model. The extreme reduction in the risk-free rate in the disaster state arises from the minimalistic model specification, in which the agent exhibits extreme saving behavior in response to a disastrous contraction. The average yield curve and the yield curves conditional on the expansion and recession states slope downward, whereas only the disaster-state yield curve slopes upward. This result suggests that a -20% contraction in the disaster state seems so severe that it blurs the distinction between the recession and expansion states.

The focus is the term structure during expansions, which carries a greater weight in determining the slope of the average yield curve owing to the associated transition probability. The 3-state model treats disasters as a state; hence, a disaster never occurs in an expansion. Consequently, when the economy enters an expansion state, the certainty equivalent of future consumption is high and the agent's marginal utility decreases, thereby increasing the risk-free rate and causing the term structure to slope downward. This result is consistent with the dominance of the consumption smoothing effect in the prior analysis. By contrast, our study models disasters as risks that are "independent of state-switching," meaning that disasters can occur even in an expansion state. Consequently, conditional on an expansion state, although the certainty equivalent of future consumption is high, the existence of disaster risk creates the possibility of reaching the disappointment threshold. This, in turn, elevates marginal utility, and hence, reduces the risk-free rate. This is a crucial mechanism that strengthens precautionary savings and helps explain the upward-sloping term structure.

Comparing the models across different disappointment thresholds and DA preferences provides deeper insights. In DA preferences, the disappointment threshold aligns with the certainty equivalent (i.e., $\delta = 1$). The comparison is illustrated in Figure 5. The choice of disappointment threshold parameter plays a crucial role in shaping the bond yield curves. Setting the disappointment threshold to 0.8 (i.e., 0.8 times the certainty equivalent) generates the highest term spreads. However, increasing the threshold to 0.83 (i.e., 0.83 times the certainty equivalent) cuts the term spreads by half. If the threshold is set equal to the certainty equivalent (as in DA preferences), it nearly eliminates the term spreads. A threshold greater than 1 ($\theta = 1.03$) has an effect similar to that of DA preferences. This finding shows that a higher threshold causes disastrous outcomes to fall below the threshold in both expansion and recession states, thereby reducing the divergence in the anticipation of disappointment between these states. Conversely, lowering the disappointment threshold parameter to 0.7 results in negative term spreads. This result resembles the outcomes in the Epstein-Zin model, where the disappointment threshold reaches zero, and suggests that for a very low threshold, disastrous outcomes will not fall below it in either expansion or recession states. Consequently, disappointment aversion is never triggered, reducing the model to conventional Epstein-Zin preferences.

[7] Dolmas (2013) analyzes the time-additive DA preferences.

Figure 5. This figure presents the real bond yield spreads with a disappointment threshold of 0.7, 0.8, 0.83, and 1.03 in GDA preferences, DA preferences, and Epstein-Zin preferences. The solid squares represent the data sample average.



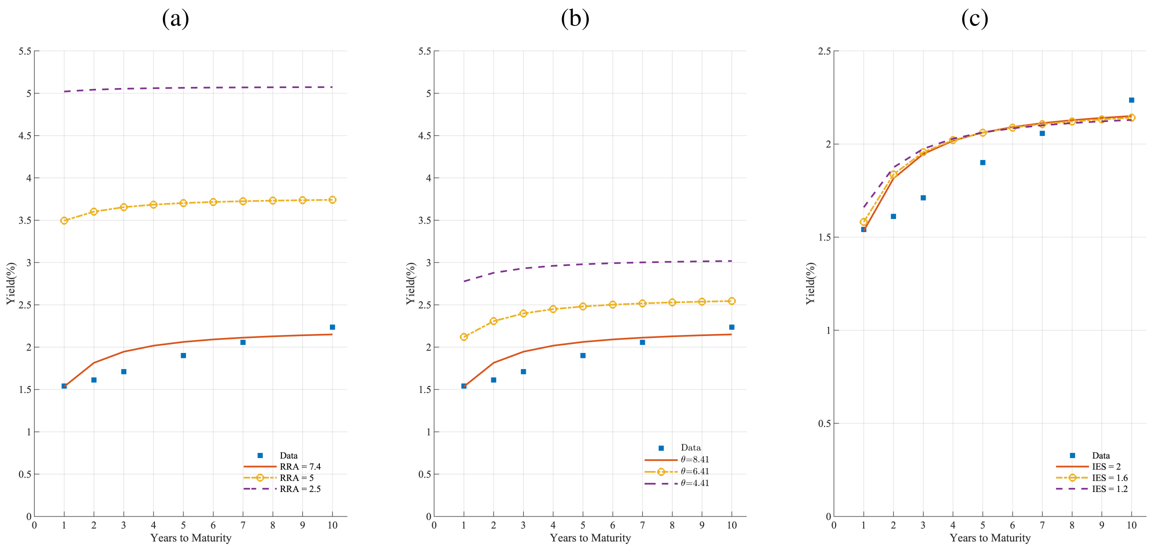
4.3 Risk Aversion, Disappointment Aversion, and IES

This section illustrates the effects of altering the RRA, IES and utility penalty parameters. As illustrated in Figure 6 (a), lowering the RRA leads to a higher average yield. This is due to the influence of risk aversion on the precautionary saving motivation channel. Reduced risk aversion dampens the agent's inclination to save for disaster risk during expansions, ultimately challenging the dominance of the precautionary saving motivation (i.e., when RRA = 2.5) and causing the term spreads to disappear.

Figure 6 (b) indicates that altering the disappointment aversion utility penalty parameter (utility penalty) significantly affects the yield curve levels. The operation of disappointment aversion in the precautionary saving motivation channel, akin to risk aversion, becomes evident. Lowering the parameter decreases the agent's fear and diminishes the need for precautionary savings during expansions.

Moreover, the intertemporal elasticity of substitution affects the intertemporal consumption smoothing channel. A lower IES parameter signifies the agent's greater reluctance to alter his/her consumption patterns, intensifying the motivation for intertemporal consumption smoothing. This increases the risk-free rate and reduces the steepness of the average yield curve (see Figure 6 (c)).

Figure 6. This figure illustrates the effect of altering the RRA, the disappointment aversion, and the IES parameters on real bond yields, respectively.



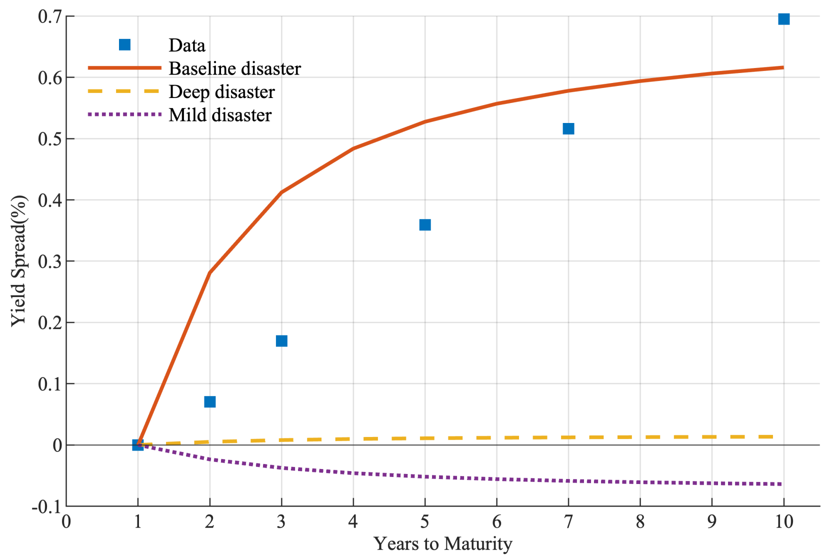
4.4 Probability Distribution of Disaster

This section examines the effects of modifying the probability distributions of rare disasters. In GDA preferences, the disappointment threshold is determined recursively. This allows for changes in the disaster probability distribution or adjustments in the weight of disaster risks relative to the Markov switching process. This may influence the certainty equivalent and shifts the disappointment threshold. It is logical to assume that severe economic contractions have lower probabilities of occurrence (deeper economic contractions are rarer events).[8] First, we increase the probability of a disaster by 1% and reduce the contraction size by 10%; this is referred to as a "mild disaster" setting. Then, we decrease the probability of a disaster by 1% and increase the contract size by 10%, referred to as the "deep disaster" setting.

Figure 7 illustrates the yield spreads for baseline, mild, and deep disaster settings. The baseline disaster setting, characterized by a -20% economic contraction with a probability of 3.5%, generates the highest yield spreads. Conversely, the yield spreads became negative in the mild disaster case, whereas they exhibit a slight upward slope in the deep disaster case. This result suggests that neither mild nor deep disaster cases led to shifts in disappointment thresholds beyond the positions associated with disastrous outcomes. Consequently, GDA preferences do not work as effectively as in the baseline

[8]Holding either the parameter of the disaster probability or the disaster size constant and modify the other can be viewed as an intermediate scenario between the previously examined non-disaster model and the disaster model.

Figure 7. This figure compares the real yield spreads across three disaster settings: baseline, mild, and deep. The baseline disaster probability is 3.5%, with an economic contraction size of -20%. In the mild disaster setting, the disaster probability is 4.5%, and the contraction size is -10%. In the deep disaster setting, the disaster probability is 2.5%, and the contraction size is -30%. The solid squares denote the sample average of data.



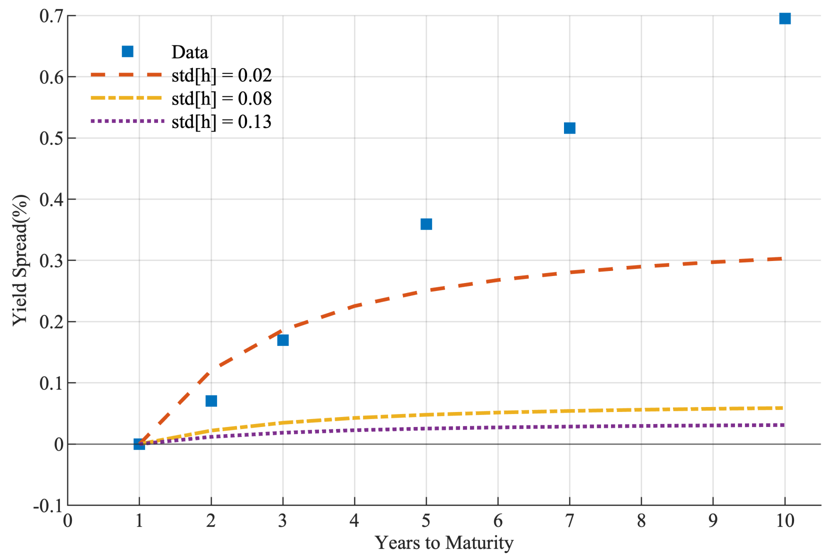
model.

Indeed, the mild disaster setting is similar to the historical occurrence of the Great Depression in the U.S. during the 1929-1933 period. However, this event is well beyond the scope of our data sample, and long-term real bond data are unavailable for such an early period. Conversely, a -30% economic contraction signifying a deeper disaster has no historical precedent in the U.S., indicating that such an extreme event is improbable for the agent to consider seriously.

Furthermore, we investigate a more general probability distribution for modeling disaster risk, such as a Gamma distribution.[9] An increased standard deviation in the Gamma distribution of disaster risk flattens the average yield curve, as illustrated in Figure 8. A higher standard deviation contributes to more disastrous outcomes that fall below the disappointment threshold, particularly in recessions. Consequently, the divergence in the agent's precautionary saving motivation between expansion and recession states diminishes, leading to a reduction in long-term bond spreads.

[9] Appendix A.1.2 provides the calculation of Gamma distribution disaster.

Figure 8. This figure compares the real yield spreads with different standard deviation of Gamma distribution for disaster risk. The mean value of disastrous contraction is set to -20%. The solid squares denote the sample average of data.



5 Conclusion

This study introduces an endowment economy model featuring a representative agent with GDA preferences to explain U.S. real bond term spread. The model combines a Markov switching process for economic fluctuations with disaster risk, which remains independent of these fluctuations. We mainly focus on unraveling the term premium puzzle, which is linked to the agent's underlying saving and borrowing behaviors. We explain the interaction between the two channels that affect the dynamics of the stochastic discount factor: the intertemporal consumption smoothing motivation and the precautionary saving motivation.

With disaster risk independence and a well-calibrated disappointment threshold, our model's stochastic discount factors are procyclical. This indicates that, even during economic expansion, the agent chooses to save. Conversely, during recessions, an agent's concerns about future disappointment diminish, leading to the propensity to borrow. Consequently, our model effectively addresses the term premium puzzle while aligning the risk-free rate with empirical data. Moreover, we address the equity premium and equity volatility puzzles.

A Appendix

A.1 Calculation of GDA preferences

Suppose the current state is $s_t = 0$. Equation (6) is expressed as

$$m(0) = \left[\frac{E_t \left[\{v(s_{t+1})\}^{1-\gamma} e^{(1-\gamma)g_{t+1}} \left(1 + \theta \mathbb{1} \left\{ g_{t+1} \le \ln \frac{\delta m(0)}{v(s_{t+1})} \right\}\right) \mid s_t = 0 \right]}{1 + \theta \delta^{1-\gamma} E_t \left[\mathbb{1} \left\{ g_{t+1} \le \ln \frac{\delta m(0)}{v(s_{t+1})} \right\} \mid s_t = 0 \right]} \right]^{\frac{1}{1-\gamma}} \tag{A1}$$

The expectation in the denominator of equation (A1) is

$$\begin{aligned}
&E_t \left[\mathbb{1} \left\{ g_{t+1} \le \ln \frac{\delta m(0)}{v(s_{t+1})} \right\} \mid s_t = 0 \right] \\
&= (1 - q_{01}) E_t \left[\mathbb{1} \left\{ g_{t+1} \le \ln \frac{\delta m(0)}{v(0)} \right\} \mid s_t = 0, s_{t+1} = 0 \right] \\
&\quad + q_{01} E_t \left[\mathbb{1} \left\{ g_{t+1} \le \ln \frac{\delta m(0)}{v(1)} \right\} \mid s_t = 0, s_{t+1} = 1 \right] \\
&= (1 - q_{01}) \left[(1-p) \Phi \left(\frac{\ln \frac{\delta m(0)}{v(0)} - \mu_0}{\sigma} \right) + p \times \text{Prob} \left\{ h_{t+1} \ge \ln \frac{v(0)}{\delta m(0)} \right\} \right] \\
&\quad + q_{01} \left[(1-p) \Phi \left(\frac{\ln \frac{\delta m(0)}{v(1)} - \mu_0}{\sigma} \right) + p \times \text{Prob} \left\{ h_{t+1} \ge \ln \frac{v(1)}{\delta m(0)} \right\} \right],
\end{aligned} \tag{A2}$$

where $\Phi(\cdot)$ denotes the c.d.f. of standard normal distribution.

The numerator of equation (A1) is

$$\begin{aligned}
&E_t \left[\{v(s_{t+1})\}^{1-\gamma} e^{(1-\gamma)g_{t+1}} \left(1 + \theta \mathbb{1} \left\{ g_{t+1} \le \ln \frac{\delta m(0)}{v(s_{t+1})} \right\} \right) \mid s_t = 0 \right] \\
&= (1 - q_{01}) \{v(0)\}^{1-\gamma} E_t \left[e^{(1-\gamma)g_{t+1}} \left(1 + \theta \mathbb{1} \left\{ g_{t+1} \le \ln \frac{\delta m(0)}{v(0)} \right\} \right) \mid s_t = 0, s_{t+1} = 0 \right] \\
&\quad + q_{01} \{v(1)\}^{1-\gamma} E_t \left[e^{(1-\gamma)g_{t+1}} \left(1 + \theta \mathbb{1} \left\{ g_{t+1} \le \ln \frac{\delta m(0)}{v(1)} \right\} \right) \mid s_t = 0, s_{t+1} = 1 \right] \\
&= (1 - q_{01}) \{v(0)\}^{1-\gamma} \left\{ (1-p) \left[e^{(1-\gamma)\mu_0 + \frac{1}{2}(1-\gamma)^2\sigma^2} \left(1 + \theta \Phi \left(\frac{\ln \frac{\delta m(0)}{v(0)} - \mu_0 - (1-\gamma)\sigma^2}{\sigma} \right) \right) \right] \right. \\
&\qquad \left. + p \left[\varphi_h(\gamma - 1) + \theta \int_{\ln \frac{v(0)}{\delta m(0)}}^{\infty} e^{(\gamma-1)h} \xi(h) dh \right] \right\} \\
&\quad + q_{01} \{v(1)\}^{1-\gamma} \left\{ (1-p) \left[e^{(1-\gamma)\mu_0 + \frac{1}{2}(1-\gamma)^2\sigma^2} \left(1 + \theta \Phi \left(\frac{\ln \frac{\delta m(0)}{v(1)} - \mu_0 - (1-\gamma)\sigma^2}{\sigma} \right) \right) \right] \right. \\
&\qquad \left. + p \left[\varphi_h(\gamma - 1) + \theta \int_{\ln \frac{v(1)}{\delta m(0)}}^{\infty} e^{(\gamma-1)h} \xi(h) dh \right] \right\}.
\end{aligned} \tag{A3}$$

where ξ_h is the p.d.f of disaster h_t and $\phi_h(a) = E(e^{ah})$ is the moment generating function.

When the current state is $s_t = 1$,

$$m(1) = \left[\frac{E_t\left[\{v(s_{t+1})\}^{1-\gamma} e^{(1-\gamma)g_{t+1}} \left(1 + \theta \mathbb{1}\left\{g_{t+1} \le \ln \frac{\delta m(1)}{v(s_{t+1})}\right\}\right) \mid s_t = 1\right]}{1 + \theta\delta^{1-\gamma} E_t\left[\mathbb{1}\left\{g_{t+1} \le \ln \frac{\delta m(1)}{v(s_{t+1})}\right\} \mid s_t = 1\right]} \right]^{\frac{1}{1-\gamma}} \tag{A4}$$

can be expressed similarly to $m(0)$.

m(0) and m(1) are solved numerically using the following procedure:

1. Conjecture the initial values of $\{m(0), m(1)\}$.

2. Calculate the right-hand side of equations (A1) and (A4) by inserting $m(0) = m(0)^0$,$m(1) = m(1)^0$, and set the resulting values of the left-hand side of equations (A1) and (A4) for $m(0)^1 = m(0)$, $m(1)^1 = m(1)$.

3. Iterate the previous procedure by calculating the right-hand side of equations (A1) and (A4) using $m(0) = m(0)^k$,$m(1) = m(1)^k$, and setting the resulting values of the left-hand side of equations (A1) and (A4) for $m(0)^{k+1} = m(0)$, $m(1)^{k+1} = m(1)$ for $k = 1, 2, \cdots$.

4. If the iteration converges; that is, $m(0)^{k+1} = m(0)^k$, $m(1)^{k+1} = m(1)^k$, then $\left\{m(0)^{k+1}, m(1)^{k+1}\right\}$ is the solution to $\{m(0), m(1)\}$.

A.1.1 When h_{t+1} is a constant

When h_{t+1} is a constant; that is, $h_{t+1} = z(z > 0)$,

in equation (A2),

$$\text{Prob}\left\{h_{t+1} \ge \ln \frac{v(s_{t+1})}{\delta m(s_t)}\right\} = \mathbb{1}\left\{z \ge \ln \frac{v(s_{t+1})}{\delta m(s_t)}\right\},$$

and in equation (A3),

$$\left[\varphi_h(\gamma - 1) + \theta \int_{\ln \frac{v(s_{t+1})}{\delta m(s_t)}}^{\infty} e^{(\gamma-1)h} \xi(h) dh\right] = e^{(\gamma-1)z}\left(1 + \theta \mathbb{1}\left\{z \ge \ln \frac{v(s_{t+1})}{\delta m(s_t)}\right\}\right).$$

A.1.2 When h_{t+1} follows a Gamma Distribution

When $h_{t+1} \sim$ i.i.d.Gamma(λ, κ), its p.d.f. is

$$\xi(h; \lambda, \kappa) = \frac{\lambda^\kappa}{\Gamma(\kappa)} h^{\kappa-1} e^{-\lambda h}, h > 0,$$

where $\Gamma(\cdot)$ is the Gamma function, and $\lambda, \kappa > 0$ are parameters determined by the expectation and variance of h_{t+1} as E$(h_{t+1} = \frac{\kappa}{\lambda}$ and Var$(h_{t+1}) = \frac{\kappa}{\lambda^2}$, respectively .

The c.d.f. of h_{t+1} is

$$\Xi(x; \lambda, k) = \int_0^x \xi(h, \lambda, k) dh. \tag{A5}$$

The moment generating function of Gamma(λ, κ) is

$$\varphi_h(a; \lambda, k) = E\left(e^{ah}\right) = \left(\frac{\lambda}{\lambda - a}\right)^k. \tag{A6}$$

In equation (A2),

$$\text{Prob}\left\{h_{t+1} \geq \ln \frac{v(s_{t+1})}{\delta m(s_t)}\right\} = 1 - \Xi\left(\ln \frac{v(s_{t+1})}{\delta m(s_t)}; \lambda, \kappa\right),$$

and in equation (A3),

$$\varphi_h(\gamma - 1) = \left(\frac{\lambda}{1 - \gamma + \lambda}\right)^\kappa,$$

$$\begin{aligned}\int_{\ln \frac{v(s_{t+1})}{\delta m(s_t)}}^{\infty} e^{(\gamma-1)h}\xi(h)dh &= \int_{\ln \frac{v(s_{t+1})}{\delta m(s_t)}}^{\infty} \frac{\lambda^\kappa}{\Gamma(\kappa)} h^{\kappa-1} e^{-(1-\gamma+\lambda)h} dh \\ &= \left(\frac{\lambda}{1-\gamma+\lambda}\right)^\kappa \left\{1 - \Xi\left(\ln \frac{v(s_{t+1})}{\delta m(s_t)}; 1-\gamma+\lambda, \kappa\right)\right\}.\end{aligned}$$

A.2 Equity Price

Equation (14)

$$\begin{aligned}\frac{P}{D}(s_t) &= E_t\left[\pi_{t+1} e^{d_{t+1}}\left(1 + \frac{P}{D}(s_{t+1})\right)\right] \\ &= E_t\Bigg[\beta e^{d_{t+1}-\gamma g_{t+1}} \left\{\frac{v(s_{t+1})}{m(s_t)}\right\}^{\frac{1}{\rho}-\gamma} \left(\frac{1 + \theta \mathbb{1}\{g_{t+1} \leq \ln \frac{\delta m(s_t)}{v(s_{t+1})}\}}{1 + \theta \delta^{1-\gamma} E_t\left[\mathbb{1}\left\{g_{t+1} \leq \ln \frac{\delta m(s_t)}{v(s_{t+1})}\right\}\right]}\right) \\ &\qquad \left(1 + \frac{P}{D}(s_{t+1})\right)\Bigg]\end{aligned} \tag{A7}$$

can be calculated in a manner similar to $\{m(0), m(1)\}$ whereas the expectation part of the numerator is

$$\begin{aligned}&E_t\left[e^{d_{t+1}-\gamma g_{t+1}} \mathbb{1}\left\{g_{t+1} \leq \ln \frac{\delta m(s_t)}{v(s_{t+1})}\right\}\right] \\ &= E_t\left[e^{\sigma_{d2}\eta_{t+1}}\right] E_t\left[e^{(1-\gamma)\mu(s_t)+(\sigma_{d1}-\gamma\sigma)\varepsilon_{t+1}} \mathbb{1}\left\{\varepsilon_{t+1} \leq \frac{\ln \frac{\delta m(s_t)}{v(s_{t+1})} - \mu(s_t)}{\sigma}\right\}\right] \\ &= e^{(1-\gamma)\mu(s_t)+\frac{1}{2}[\sigma_{d2}^2+(\sigma_{d1}-\gamma\sigma)^2]} \Phi\left(\frac{\ln \frac{\delta m(s_t)}{v(s_{t+1})} - \mu(s_t) - \sigma_{d1}\sigma + \gamma\sigma^2}{\sigma}\right).\end{aligned} \tag{A8}$$

B Additional Test: 3-state Model

This model considers Dolmas (2013) disaster specification. The consumption process is

$$\Delta \ln C_{t+1} = g_{t+1} = \begin{cases} \mu_0 + \sigma \varepsilon_{t+1} & \text{if} \quad s_t = 0 \\ \mu_1 + \sigma \varepsilon_{t+1} & \text{if} \quad s_t = 1 \\ -z(z > 0) & \text{if} \quad s_t = 2 \end{cases} \tag{B1}$$

where $s_t = 0, 1, 2$ refer to the expansion, recession, or disaster states, respectively. The transition matrix is

$$Q = \begin{pmatrix} (1-p)(1-q_{01}) & (1-p)q_{01} & p \\ (1-p)q_{10} & (1-p)(1-q_{10}) & p \\ 1/2 & 1/2 & 0 \end{pmatrix}.$$

The solution method is the same as that in our main model, although some calculations in AppendixA.1 differ.

For $s_t = 0$, the expectation in the denominator of equation (A1) is

$$\begin{aligned} & E_t\left[\mathbb{1}\left\{g_{t+1} \leq \ln \frac{\delta m(0)}{v\left(s_{t+1}\right)}\right\} \mid s_t = 0\right] \\ =&(1-p)\left(1-q_{01}\right) E_t\left[\mathbb{1}\left\{g_{t+1} \leq \ln \frac{\delta m(0)}{v(0)}\right\} \mid s_t = 0, s_{t+1} = 0\right] \\ & + (1-p)q_{01} E_t\left[\mathbb{1}\left\{g_{t+1} \leq \ln \frac{\delta m(0)}{v(1)}\right\} \mid s_t = 0, s_{t+1} = 1\right] \\ & + p E_t\left[\mathbb{1}\left\{g_{t+1} \leq \ln \frac{\delta m(0)}{v(2)}\right\} \mid s_t = 0, s_{t+1} = 2\right] \\ =&(1-p)\left(1-q_{01}\right) \Phi\left(\frac{\ln \frac{\delta m(0)}{v(0)} - \mu_0}{\sigma}\right) + (1-p)q_{01} \Phi\left(\frac{\ln \frac{\delta m(0)}{v(1)} - \mu_0}{\sigma}\right) + p \times \Phi\left(\frac{\ln \frac{\delta m(0)}{v(2)} - \mu_0}{\sigma}\right), \end{aligned} \tag{B2}$$

the numerator of equation (A1) is

$$\begin{aligned}
&E_t\left[\{v(s_{t+1})\}^{1-\gamma} e^{(1-\gamma)g_{t+1}}\left(1+\theta\mathbb{1}\left\{g_{t+1}\le \ln\frac{\delta m(0)}{v(s_{t+1})}\right\}\right) \mid s_t=0\right]\\
=&(1-p)(1-q_{01})\{v(0)\}^{1-\gamma}E_t\left[e^{(1-\gamma)g_{t+1}}\left(1+\theta\mathbb{1}\left\{g_{t+1}\le \ln\frac{\delta m(0)}{v(0)}\right\}\right) \mid s_t=0, s_{t+1}=0\right]\\
&+(1-p)q_{01}\{v(1)\}^{1-\gamma}E_t\left[e^{(1-\gamma)g_{t+1}}\left(1+\theta\mathbb{1}\left\{g_{t+1}\le \ln\frac{\delta m(0)}{v(1)}\right\}\right) \mid s_t=0, s_{t+1}=1\right]\\
&+p\{v(2)\}^{1-\gamma}E_t\left[e^{(1-\gamma)g_{t+1}}\left(1+\theta\mathbb{1}\left\{g_{t+1}\le \ln\frac{\delta m(0)}{v(2)}\right\}\right) \mid s_t=0, s_{t+1}=2\right]\\
=&(1-p)(1-q_{01})\{v(0)\}^{1-\gamma}e^{(1-\gamma)\mu_0+\frac{1}{2}(1-\gamma)^2\sigma^2}\left[1+\theta\Phi\left(\frac{\ln\frac{\delta m(0)}{v(0)}-\mu_0-(1-\gamma)\sigma^2}{\sigma}\right)\right]\\
&+(1-p)q_{01}\{v(1)\}^{1-\gamma}e^{(1-\gamma)\mu_0+\frac{1}{2}(1-\gamma)^2\sigma^2}\left[1+\theta\Phi\left(\frac{\ln\frac{\delta m(0)}{v(1)}-\mu_0-(1-\gamma)\sigma^2}{\sigma}\right)\right]\\
&+p\{v(1)\}^{1-\gamma}e^{(1-\gamma)\mu_0+\frac{1}{2}(1-\gamma)^2\sigma^2}\left[1+\theta\Phi\left(\frac{\ln\frac{\delta m(0)}{v(2)}-\mu_0-(1-\gamma)\sigma^2}{\sigma}\right)\right].
\end{aligned} \tag{B3}$$

The case of $s_t = 1$ is similar to that of $s_t = 0$. For $s_t = 2$, the expectation in the denominator is

$$E_t\left[\mathbb{1}\left\{g_{t+1}\le \ln\frac{\delta m(0)}{v(s_{t+1})}\right\} \mid s_t=2\right]=\frac{1}{2}\mathbb{1}\left\{z\ge \ln\frac{v(0)}{\delta m(2)}\right\}+\frac{1}{2}\mathbb{1}\left\{z\ge \ln\frac{v(1)}{\delta m(2)}\right\}, \tag{B4}$$

and the numerator is

$$\begin{aligned}
&E_t\left[\{v(s_{t+1})\}^{1-\gamma} e^{(1-\gamma)g_{t+1}}\left(1+\theta\mathbb{1}\left\{g_{t+1}\le \ln\frac{\delta m(0)}{v(s_{t+1})}\right\}\right) \mid s_t=2\right]\\
=&\frac{1}{2}\times\{v(0)\}^{1-\gamma}e^{(1-\gamma)(-z)}\mathbb{1}\left\{z\ge \ln\frac{v(0)}{\delta m(2)}\right\}+\frac{1}{2}\times\{v(1)\}^{1-\gamma}e^{(1-\gamma)(-z)}\mathbb{1}\left\{z\ge \ln\frac{v(1)}{\delta m(2)}\right\}.
\end{aligned} \tag{B5}$$

Applying the numerical method in AppendixA.1, we obtain $m(0), m(1), m(2)$ in the 3-state model and calculate the stochastic discount factors and bond prices.

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