Review of Economic Analysis 17 (2025) 345–378 1973-3909/2025345

Value of Bidders Default Risk Information: Designing Reliable Auction Using Post-Bidding Mechanism

ANIRBAN CHATTERJEE*
Indian Institute of Management, Shillong, India

SUBHADIP MUKHERJEE
Indian Institute of Management, Shillong, India

Traditional auction theory typically assumes that bidders will fully honor their commitments upon winning. However, this assumption often proves inadequate in high-stakes auctions, such as those for infrastructure projects or luxury assets, where securing the necessary funds post-auction can pose challenges. Factors such as market volatility, liquidity constraints, or delays in financing frequently result in bidder defaults, leading to substantial disruptions for sellers, including the costs of re-auctioning and project delays. This paper seeks to bridge this gap in auction theory by exploring how comprehensive information about bidders' default risks can be utilized through a straightforward post-bidding mechanism. Furthermore, it highlights the advantages this approach offers to the auctioneer compared to scenarios where such critical information is unavailable.

Keywords: Auction, Default, Reliability

1 Introduction

Auction theory has long been a central framework in economics, particularly for pricing and allocating goods in competitive markets. It underpins key mechanisms in business transactions, such as sales of assets, government contracts, and procurement processes, with the goal of optimizing revenue or ensuring efficient resource allocation. Traditional auction models, such as the first-price and second-price sealed bid auctions, assume that bidders are rational, have perfect information, and will fulfill their financial commitments once they win the auction. These models typically maximize expected revenues based on bidders' valuations and bidding strategies, operating under the presumption of stable market conditions and reliable payments. However, the limitations of this presumption motivate the present study.

*Corresponding author: anirban.phdwp23@iimshillong.ac.in

1.1 Research Motivation and Research Gap

However, in high-value auctions, especially in volatile markets, these assumptions may break down. Market fluctuations, liquidity constraints, credit risk, and the potential for strategic miscalculations can all lead to a bidder's inability to secure the necessary funds to honor their bid. For instance, a sudden downturn in the market or unexpected regulatory changes may impair a bidder's financing options, rendering them unable to follow through on the commitment. In such scenarios, even though the auction is designed to allocate the asset to the highest bidder, external factors and strategic bidding behavior often result in defaults, undermining the auction's intended outcomes. While a bid is typically regarded as a legally binding contract, enforcing it in practice can be difficult for sellers. Legal actions to compel bidders to honor their financial commitments are often prolonged and prohibitively expensive [Pacific Debt Relief, 2023]. Weak enforcement of contractual obligations or declaration of bankruptcy by the winning bidder aggravate the issue further [Iyer and Schoar, 2024, Board, 2007]. A notable example [Gupta, 2008] of a high-value auction in the infrastructure sector where a winning bidder withdrew due to an inability to arrange funds is found in India's national highway construction projects. Instances like these often involve complex bidding processes facilitated by the National Highways Authority of India (NHAI). In one such scenario, a bidder who had initially secured a significant contract later defaulted. The reasons included the inability to meet the financial criteria and challenges in arranging funds amidst volatile economic conditions. This is not uncommon in infrastructure projects, especially those requiring massive capital investments. Bidders might overestimate their financial capacity or face unforeseen challenges such as increased borrowing costs, liquidity crunches, or changes in market conditions. These defaults can significantly delay project timelines and increase administrative burdens for entities such as NHAI, which must re-tender the projects or negotiate with alternative bidders. Therefore, the concept of opportunity cost of the seller is pivotal in auction theory for analyzing the economic impacts of bidder defaults. A default results in more than just immediate revenue loss; it imposes implicit costs tied to foregone alternatives. For example, prioritizing a slightly lower bid from a reliable bidder could ensure timely execution and efficient utilization of resources. This is especially crucial in industries like infrastructure, energy, or public procurement, where defaults can lead to significant delays, cascading inefficiencies, and disruptions to planned operations. In public auctions, the legal expenses incurred by the government to address defaults divert resources from other essential areas, thereby adding to the overall fiscal burden. Mitigating these challenges is key to developing reliable auction mechanisms that strike a balance between maximizing revenue, ensuring reliability, and maintaining efficiency.

1.1.1 Research Gap

In traditional auction models, it is assumed that once a bid is placed, the bidder is committed to the transaction. Models like first-price auctions and second-price auctions primarily focus on the distribution of bids and the price realization, assuming that the winner will pay the price they bid. However, this assumption often breaks down in high-stakes auctions, particularly those involving large infrastructure projects, real estate, or government contracts. In such auctions, bidders may be unable to arrange the necessary financing to complete the deal, even after being declared the winner, thus leading to auction failure and delays in the awarding of contracts.

This issue of bidder withdrawal due to financing constraints remains largely under-explored in auction theory. Existing research, such as [Melnik and Alm, 2002, Krishna, 2003], touches on the general structure of auction models but does not consider the implications of bidder defaults. Some studies do explore bidder behavior under uncertainty or financial constraints, but these models rarely apply to the high-value contexts where default risks are more pronounced and caused by exogenous factors ensuring that those uncertainties are beyond the control of the bidders. Moreover, when considering collusion or strategic manipulation, the traditional models fail to address how bidders might intentionally withdraw after securing the win in order to re-negotiate terms or re-enter the auction when conditions are more favorable [Che and Gale, 1998]. Lu [2009] examines scenarios where bidders face an opportunity cost when deciding to place a bid. Lamping [2007], Lee and Li [2019] examine auctions with bidder defaults and analyzes their impact on overall welfare. However, the study assumes that bidders have control over their default probabilities, which can be reduced through the imposition of penalty clauses. However, when bidders' default probabilities are beyond their control as they are driven by exogenous factors, imposing penalties for defaults exceeding a certain threshold discourages participation, ultimately reducing the expected revenue. Dirk Engelmann and Valente [2023] partially explore this problem by analyzing auctions with second-chance offers; however, it neither considers the costs incurred by the seller due to the default of the initially declared winner nor focuses on designing a reliable auction mechanism that minimizes the probability of default of the initially declared winner. These gaps lead to a natural design question: *Can we frame a reliability-aware auction as a traditional optimal auction when the winning bidder may, with an exogenously determined finite probability, fail to honor the bid?* Our position is that a classic optimal-auction approach may not be a natural fit here. Once a winner can default and the seller bears opportunity-cost losses (delay, foregone alternatives, reallocation effort), expected revenue depends on award, execution, and post-default contingencies, making the problem dynamic and state-contingent rather than one-shot. This undermines the clean, single-dimensional structure behind standard results with monotone allocation and the payment identity, especially when penalties are bounded and reliability differs across bidders or is only partially observed—effectively yielding multi-dimensional bidder types. Allowing backup allocation can improve allocative efficiency, but it does not minimize the probability that the first winner defaults, and it leaves the seller exposed to opportunity-cost losses when defaults occur. Taken together, these features render general closed-form “optimal auction” characterizations fragile and motivate a focus on reliability-aware and implementable designs with reliability-weighted selection and on benchmarking their performance against idealized upper bounds.

Consideration of reliability in the context of auction has significant practical significance. In the absence of reliable auction designs, sellers in high-value auctions are exposed to considerable risks, including revenue loss and delays in project execution. These risks can render the auction process ineffective, particularly in scenarios characterized by financial uncertainty. For example, infrastructure projects like road construction or power plant development, which demand substantial capital investment, may experience bidder defaults due to unexpected financing challenges. Such defaults often necessitate expensive re-auctions and increase the likelihood of revenue shortfalls for the seller. However, the question remains whether having complete information about bidders’ default risks caused by exogenous factors provides a tangible advantage in designing reliable auctions compared to scenarios where this information is unavailable to the seller.

1.2 Our Contribution

We propose a post-bidding mechanism that prioritizes highly reliable bidders over unreliable ones, while ensuring that the expected revenue remains unaffected. Minchuk and Sela [2018] introduce a pre-bidding auction mechanism but assumes that all bidders are perfectly reliable. We extend this approach by incorporating bidders’ default risk information into the auction framework, ensuring that a reliable bidder wins the auction with certainty, provided his bidding capacity is competitive relative to the unreliable bidders. Our contributions are as follows:

Contribution 1. We introduce a sequential mechanism for high-value auctions, accommodating scenarios where all but one bidder may withdraw their bids due to external influences, such as market volatility caused by macroeconomic shocks. Our analysis reveals that the proposed mechanism exhibits lower sensitivity to bidders’ default probabilities compared to the conventional second-price auction.

Contribution 2. We establish the condition under which incorporating risk information into the auction mechanism guarantees to yield higher expected revenue with significantly large number of unreliable bidders compared to a standard second-price auction, where such risk information is unavailable to the seller.

Contribution 3. In the specific scenario where bidders’ private values are drawn from a uniform distribution, we prove the existence of a threshold default probability beyond which integrating risk information into the auction mechanism yields superior outcomes compared to scenarios where such risk information is absent, regardless of the number of unreliable bidders. We computed the values of the threshold default probability for different situations.

The structure of the paper is as follows: Section 2 presents a concise overview of the related literature. Section 3 details the definitions and the model. Section 4 provides an in-depth analysis, while Sections 5 and 6 discuss policy implications and future research directions, respectively.

2 Related Literature

This paper is related to several key research areas:

2.1 Auctions with Reputation Systems

The closest research theme to this paper pertains to auctions involving reputation systems. The integration of reputation mechanisms into auctions, particularly regarding sellers, is a well-established field. Experimental research, such as [Bolton et al., 2002], highlights that reputation systems can significantly enhance the success rates of transactions. Field studies similarly underscore the positive effects of seller reputation on auction outcomes [Ba and Pavlou, 2002, Melnik and Alm, 2002, Dellarocas and Wood, 2008, Lucking-Reiley et al., 2007]. For instance, Livingston [2005] investigates the impact of a robust reputation in online auction settings. In contrast, Kas et al. [2023] challenge the conventional view in economic sociology that markets require institutional frameworks to prevent opportunism. They argue that, while reputation systems are effective in identifying trustworthy sellers, the presumption that they are necessary to combat dishonesty reflects an unsubstantiated cynicism about human behavior. Gregg and Scott [2006] examine the role of online reputation systems in helping buyers avoid fraudulent auctions.

In contrast, the exploration of buyer reputation systems in auction settings is relatively underdeveloped. Tadelis [2016] demonstrates that positive ratings from buyers can aid their entry into the auction market as sellers. Cabral and Hortacsu [2010] provide evidence from eBay suggesting that buyer ratings can influence seller behavior, although this becomes problematic when separate ratings exist for buyers and sellers, leaving the relevance of buyer ratings in the seller context ambiguous. Dirk Engelmann and Valente [2023] delve into buyer reputation systems, focusing on how second-chance offers affect bidding strategies and how habitual defaulters exploit these mechanisms. Their experimental results underscore the importance of buyer ratings, revealing their influence on bidding behavior and the strategic interactions shaped by competing buyers' ratings. However, the study does not propose a mechanism to prioritize reliable bidders or address the seller's costs due to bidder defaults.

2.2 Auctions with Default Risks

This paper builds upon research exploring auctions where bidders or auctioneers might withdraw strategically. For example, Roelofs [2002] investigates a common-value auction model that explicitly allows bidder defaults. Pagnozzi and Saral [2019] discuss an auction mechanism that accommodates bidders with limited liability, enabling them to mitigate ex-post losses through default or resale options. Harstad and Rothkopf [1995] explore the use of withdrawable bids as a strategy to counter the winner's curse, proposing penalties to deter such behavior. Similarly, Lorentziadis [2014] studies sealed-bid auctions where the auctioneer might default with some probability, while ChienHsing Wu and Ho [2022] empirically examine the behavior of non-paying bidders. Holland and O'Sullivan [2007] address the *bid taker's exposure problem* in combinatorial auctions, while Lorentziadis [2014] discusses the costs incurred by the seller when a winning bidder defaults. Lee and Li [2019] look at the role of costly participation in auctions, exploring the effects of bidder defaults on social welfare. Chillemi and Galavotti [2024] focus on scenarios where firms with cheaper technologies are more prone to ex-post defaults, offering conditions that ensure a first-price auction with audit is optimal. Arve and Martimort [2024] investigate how uncertainty in the cost of add-ons introduces background risks for the auctioneer, requiring the inclusion of a risk premium in procurement auctions.

2.3 Risk Management and Credibility in Auctions

A related body of research investigates how risks are managed within auction settings. For instance, [DeMarzo et al., 2005] analyze security-bid auctions which considers informal settings in which the bidders offer arbitrary securities and the seller chooses the most attractive bid based on its beliefs ex-post. Engel and Wambach [2005] examine cases where the winning bidder experiences financial distress or bankruptcy before fulfilling their obligations, proposing mechanisms such as *split awards* and *rationing* to mitigate these risks.[Spagnolo et al., 2006] suggest average pricing mechanisms to reduce price competition and prevent cost overruns, thereby alleviating the financial pressure on bidders in volatile market conditions. This research intersects with the literature on credible auctions, particularly those concerned with the trustworthiness of sellers [Tadelis, 2016]. Theoretical studies have focused on mechanisms to build trust and address issues like moral hazard and adverse selection, aiming to achieve more efficient market outcomes [Tadelis, 2002]. Additionally, research has examined contractual mechanisms such as retainage to reduce moral hazard on the seller's side [Walker et al., 2021]. Recent studies, like [Akbarpour and Li, 2020], propose that the ascending price auction with reserves is the only credible, optimal, and strategy-proof mechanism, though it involves significant communication complexity.

Our work distinguishes itself from the aforementioned studies by introducing a sealed-bid auction mechanism that prioritizes reliable bidders. This mechanism aims to minimize the likelihood of a winning bidder defaulting while maximizing the seller's expected revenue. By attributing the risk of default to external, uncontrollable factors, the model excludes penalty-based solutions and instead focuses on optimizing the trade-off between revenue generation and enhancing auction outcome reliability, particularly by reducing the probability of a default by the winner.

3 Definitions and Model

We first introduce the following definitions before moving on to the model.

3.1 Bidder's Reliability

Bidder's reliability refers to the likelihood that a bidder will fulfill its obligations upon winning an auction. Economically, it is inversely related to the bidder's default probability (p), which quantifies the probability of failure to meet contractual commitments due to financial, operational, or external market factors. A highly reliable bidder exhibits a low value of p signifying reduced vulnerability to market volatility. Therefore, bidder's reliability, framed in terms of default probability, serves as a critical metric for understanding and managing susceptibility to market volatility in auctions. By incorporating reliability considerations, auctioneers can achieve outcomes that are not only financially optimal but also robust against external economic uncertainties. This approach aligns with the broader objectives of risk management and sustainable economic practices in auction design.

3.2 Auction Reliability

Definition 1. *An auction $\mathcal{A}_1$ is deemed more reliable than auction $\mathcal{A}_2$ if the expected probability of the first declared winner fulfilling their commitment without default in $\mathcal{A}_1$ exceeds that in $\mathcal{A}_2$.*
.

We represent the reliability of the auction $\mathcal{A}$ by $\mathcal{R}(\mathcal{A})$. Therefore, $\mathcal{R}(\mathcal{A}_1) > \mathcal{R}(\mathcal{A}_2)$ indicates that auction $\mathcal{A}_1$ is more reliable than the auction $\mathcal{A}_2$.

3.3 Seller's Opportunity Cost

We assume that the seller incurs an opportunity cost C where $0 < C < 1$ when the winning bidder defaults. We assume $0 < C < 1$ since $C \geq 1$ offers little to no incentive for the seller to conduct the auction, particularly when bidders' private valuations are drawn from any regular distribution with support in $[0, 1]$.

Refer to Appendix B 6 for detailed discussion on *Auction Reliability* and *Seller's Opportunity Cost*.

3.4 Model Specification

A single object is available for sale in an auction. The auction features n risk-neutral bidders with $n \geq 3$, among whom one bidder is completely reliable, ensuring a probability of default equal to 0 in case it wins. This bidder who never defaults is denoted as B_r. The remaining $n - 1$ bidders are subject to market volatility, which affects their ability to fulfill financial commitments after winning. Their probability of default is denoted as p, where $0 < p \leq 1$. These bidders are represented as B_u^i where $0 \leq i \leq n - 1$. It is important to note that bidders cannot influence their default probability p, as it is dictated by several exogenous factors such as prevailing market volatility and many others. All bidders independently draw their private valuations from the same cumulative distribution $F(x)$, with the corresponding probability density function $f(x)$. We assume that $f(x)$ is positive, continuous, and differentiable on the interval $[0, 1]$, and equals zero elsewhere. Furthermore, we impose the standard monotone hazard rate property on $f(x)$ which requires that $\frac{1-F(x)}{f(x)}$ is non-increasing.

3.4.1 Remarks and Assumptions

We set out a few remarks on the model's specification and the model's key assumptions below before moving on to the analysis.

Remark 1. Although penalty clauses are sometimes proposed to deter default in auctions in extant literature(e.g., [Lee and Li, 2019]), when default risk is high and exogenous they neither reduce defaults nor preserve participation. We therefore set penalties aside and focus on mechanisms that remain effective under elevated, non-controllable default probabilities.

Remark 2. Our model is based on the observations made in extant literature (such as [Sharma et al., 2014]) that most of the firms are subject to market risk and uncertainties because they operate in dynamic economic environments where external factors influence their outcomes.

Refer to Appendix 6 for a detailed discussion on the above remarks.

Assumption 1. *[Gretschko and Mass, 2024, Krishna, 2003] All unreliable bidders adopt a symmetric pure bidding strategy* $\beta : [0, 1] \to [0, 1]$*, where* $\beta(\cdot)$ *is a monotonically increasing function and* $\beta(0) = 0$*.*

In auction theory, it is commonly assumed that all bidders adhere to a symmetric bidding strategy $\beta : [0, 1] \to [0, 1]$ for expositional convenience, with $\beta(\cdot)$ being a monotonically increasing function and $\beta(0) = 0$. This assumption is justified by the fact that symmetry naturally follows when bidders are *ex-ante* identical, implying that each unreliable bidder draws their private values independently from the same distribution and faces the same set of auction rules. This symmetry ensures that no unreliable bidder has a strategic advantage, leading to the adoption of symmetric bidding strategies in equilibrium. The monotonicity of $\beta(\cdot)$ aligns with the natural outcome in first-price auctions, where bidders with higher private values place higher bids, thereby ensuring allocative efficiency. The condition $\beta(0) = 0$ ensures that a bidder with a private value of zero will bid zero, which is a rational strategy to avoid incurring a guaranteed loss in the event of winning. This assumption simplifies the analysis by reducing the complexity of bidder interactions.

Definition 2. *(Equilibrium) We define the equilibrium as a Bayesian Nash Equilibrium (BNE), where the strategy profile of unreliable bidders maximizes their expected utility by submitting bids that surpass both the bids of other unreliable bidders and the reliable bidder's private valuation. This bidding strategy is determined based on their beliefs about the private values of all participants. The equilibrium ensures that unreliable bidders optimize their probability of winning while considering the strategic interactions with both reliable and unreliable bidders.*

3.5 Auction Mechanism

The steps in the auction mechanism are as follows:

Step 1. The auctioneer requests all $n - 1$ bidders, B_u^i, whose default probability is p, to submit their bids.

Step 2. The auctioneer ranks the bids and identifies the provisional winner based on the highest bid.

Step 3. The auctioneer allows the highest and second-highest bidders to openly adjust their bids, with the process continuing iteratively until one of the bidders decides to withdraw from further raising the bid. Let's assume that b_h is the highest bid placed in this step.

Step 4. The auctioneer inquires whether bidder B_r, the bidder with a default probability of 0, is willing to pay the amount $r = (1 - p)b_h$.

Step 5. If B_r consents to pay the amount specified in *step 4*, the item is awarded to B_r. Otherwise, the item is allocated to the winning bidder from *step 3*, who is then required to pay the final bid he submitted in *step 3* i.e b_h.

The mechanism evidently provides a favorable advantage to the reliable bidder. In real-world procurement auctions, it is a common phenomenon that incumbent bidders—those with prior contracts or established relationships with the procuring entity—often receive preferential treatment [Colucci et al., 2012]. Such advantages may take different forms, including the right to match the highest competing bid, additional weighting in evaluation metrics, or exclusive rights to negotiate contract terms.

3.6 Information Structure

The model's information structure is outlined as follows:

Private Information. Each bidder's valuation is private and known solely to the respective bidder, with no access to other bidders or the seller. At *Step 4*, the reliable bidder learns the reserve price r, which remains confidential between the seller and the reliable bidder, without being shared with the unreliable bidders.

Common Knowledge. The distribution function from which bidders derive their private valuations is common knowledge to all participants, including the seller. In addition, each bidder's default probability representing its respective reliability is common knowledge as well to all bidders and the seller.

At this stage, a key question arises: as per *Step 1*, only $n - 1$ unreliable bidders are required to submit bids. *Would including all n bidders, the reliable bidder included, make the auction more competitive?* The answer is no. If the reliable bidder knows they are the preferred bidder and will always receive a second chance if he is not the highest bidder, he would strategically bid just below the second-highest bid to maintain the equilibrium. Consequently, incorporating the reliable bidder into *Step 1* would not change the auction's outcome. Moreover, some auction mechanisms allow the item to be awarded to the second-highest bidder if the winning bidder defaults [Dirk Engelmann and Valente, 2023, Bagchi et al., 2013]. In such auctions with second-chance offers, a default by the winning bidder delays payment, as the seller must engage the second-highest bidder to assess their willingness and ability to pay, and renegotiate the terms. This process prolongs the transaction, creates administrative burdens, and may further delay payment if the second bidder is hesitant or imposes additional conditions. These delays disrupt the seller's cash flow, reduce operational efficiency, and negatively affect profitability by postponing revenue realization and increasing administrative expenses [Paul et al., 2012].

4 Analysis

The seller implements the standard $\mathcal{A}^{SP}$ auction when no information about bidder risk is available. Conversely, if the seller possesses risk information about the bidders, the proposed auction mechanism 3.5, referred to as $\mathcal{A}^{P}$, is conducted. To analyze the interplay between bidder reliability and the expected revenue under the proposed mechanism, a comparison is made with the outcomes of the standard second-price auction.

Proposition 1. *(Auction Reliability) For any distribution function $f(x)$, a threshold default probability p^* for the unreliable bidders is guaranteed to exist such that for any $p > p^*$, the proposed auction mechanism $\mathcal{A}^{P}$ will consistently exhibit higher reliability compared to the* *standard second-price auction* $\mathcal{A}^{SP}$ *i.e.* $\mathcal{R}(\mathcal{A}^{P})\big|_{p\geq p^*} \geq \mathcal{R}(\mathcal{A}^{SP})\big|_{p\geq p^*}$.

The existence of a threshold default probability p^*, beyond which the proposed auction mechanism $\mathcal{A}^P$ consistently outperforms the standard second-price auction ($\mathcal{A}^{SP}$) in terms of reliability, carries significant implications in economic theory and practice. This highlights how auction mechanisms can be tailored to enhance outcomes in markets with varying levels of risk and uncertainty. The threshold p^* represents a critical point where the default risk of unreliable bidders begins to significantly influence auction outcomes. Economically, this threshold marks the limits of effectiveness for the standard second-price auction in maintaining bidder reliability. Above this threshold, $\mathcal{A}^P$ becomes superior in minimizing default risks, indicating its adaptability to high-risk environments. Winning bidder defaults can disrupt resource allocation, necessitate costly re-auctions, and delay the realization of revenues. The higher reliability of $\mathcal{A}^P$ in such scenarios ensures smoother market operations by reducing these inefficiencies. This is particularly important in high-stakes auctions such as infrastructure projects or public procurements, where defaults can have cascading economic consequences. By prioritizing mechanisms that implicitly or explicitly reduce the likelihood of defaults, $\mathcal{A}^P$ incentivizes participation from bidders who are less likely to default. This improves the probability of successful contract execution. The superiority of $\mathcal{A}^P$ beyond the threshold p^* underscores the economic value of designing auction mechanisms that account for bidder heterogeneity, especially regarding financial reliability. Such mechanisms reduce the systemic risks posed by default-prone participants, promoting overall market resilience. In sectors where bidder reliability directly affects social welfare—such as public infrastructure or defense procurement—the ability of $\mathcal{A}^P$ to outperform the standard auction format translates to broader societal benefits. Reliable auction outcomes ensure that critical projects are less likely to face delays or cost overruns due to bidder defaults. This result emphasizes the importance of tailoring auction designs to market conditions. While the second-price auction has broad theoretical appeal due to its allocative efficiency, its performance deteriorates in environments with significant bidder unreliability. Mechanisms like $\mathcal{A}^P$, which are robust to such conditions, reflect the practical need for dynamic and adaptive auction designs in real-world markets.

The central question that remains is: *Does achieving reliability in auction outcomes come at the cost of revenue?* We will address this question in the following analysis. In traditional auction theory, the opportunity cost loss of the seller is often overlooked when the winning bidder defaults. This oversight can have significant economic implications, as the seller not only faces direct revenue loss but also indirect costs arising from missed opportunities and inefficiencies in resource allocation. Opportunity cost, in economic terms, refers to the value of the next best alternative forgone due to a particular decision. In the context of auctions, when a winning bidder defaults, the seller loses the opportunity to sell the good or service to other potential bidders who might have been willing and able to complete the transaction. For example, in public infrastructure auctions, when the winning bidder defaults on payment or project execution, the government loses not only the immediate revenue but also the economic benefits of timely project completion, such as job creation, economic stimulation and overall economic sustainability [Hussain et al., 2023].

Before moving on to the next proposition and numerical analyses, let us present the following *Lemma*.

Lemma 1. *If the bidders draw their private values from any regular distribution $f(x)$ so that it satisfies $2f(x) > f'(x)$ for all $x \in [0,1]$, the expected value of b_h as per the step 3 of mechanism 3.5 in $\mathcal{A}^P$ is given by*

$$E[b_h] = (n-1)(n-2)\bigg[\int_0^1 \int_0^{g(v_1)} g(v_1)\, F^{n-3}(v_2)\, f(v_1) f(v_2)\, dv_2\, dv_1 + \int_0^1 \int_{g(v_1)}^{v_1} v_2\, F^{n-3}(v_2)\, f(v_1) f(v_2)\, dv_2\, dv_1\bigg]$$

where $g(\cdot)$ is a continuous and differentiable function and $g(x) \leq x$.

Lemma 2. *For any regular distribution function $f(x)$ with support in $[0,1]$, the second-highest valuation, $x^{(2)}$, drawn from $f(x)$ satisfies the following relation:* $\lim_{n\to\infty} x^{(2)} \approx 1$

Using *Lemma* 1 and 2, we now present the following proposition which presents the properties of the expected revenue of the proposed mechanism 3.5 in comparison with standard second-price auction:

Proposition 2. *(Expected Revenue) The following results hold for any regular distribution $f(x)$ that satisfies $2f(x) > f'(x)$ and $0 < C < 1$ where $\pi(\mathcal{A})$ denotes the expected revenue of auction $\mathcal{A}$:*

1. $\lim_{n\to\infty} \pi(\mathcal{A}^P) \geq \lim_{n\to\infty} \pi(\mathcal{A}^{SP})$
2. $\lim_{n\to\infty} \left|\frac{\partial \mathcal{A}^P}{\partial p}\right| \leq \lim_{n\to\infty} \left|\frac{\partial \mathcal{A}^{SP}}{\partial p}\right|$

When the expected revenue of the proposed auction mechanism surpasses that of the standard second-price auction for a sufficiently large number of bidders, it signals a critical economic advantage in environments with high bidder unreliability. This outcome suggests that the proposed mechanism is more robust and reliable in mitigating the adverse effects of bidder defaults on the seller's revenue. The reduced sensitivity of revenue to default probabilities implies that the seller can achieve greater stability in expected revenue, even as the proportion of unreliable bidders increases or their likelihood of default becomes significant. This resilience arises because the proposed mechanism incorporates features—such as prioritizing reliable bidders or optimizing bid allocations based on default risks—that internalize the costs associated with bidder unreliability. As a result, the seller benefits from a mechanism that not only maximizes revenue in competitive scenarios with many bidders but also offers a safeguard against the volatility induced by high default risks, enhancing overall auction efficiency and predictability.

1. Proposition 2 is guaranteed to hold universally for all distributions $f(x)$ that satisfy the condition $2f(x) > f'(x)$ for all $x \in [0, 1]$. For example, uniform distribution or truncated exponential distribution with rate parameter $\lambda > -2$ will fall under this category. However, in certain special cases the distribution function $f(x)$ might not satisfy the inequality $2f(x) > f'(x)$ all over its domain, yet Proposition 2 could still hold. For instance, if the bid submitted by the provincial winner in *Step 2* of mechanism 3.5 is denoted by b_w, and the condition $f'(b_w) < 2f(b_w)$ holds, then Proposition 2 remains valid.

2. We examine the comparison between two auction mechanism in more detail for a special case in section 4.1.

Corollary 1. *Proposition 2 is also applicable to the standard first-price auction.*

The *Revenue Equivalence Principle* [Heydenreich et al., 2009] asserts that, under the assumptions of risk-neutrality, independent private values, symmetric bidders, and efficient allocation, all standard auction formats produce the same expected revenue for the seller and the same expected utility for the bidders. Consequently, reliability is identical for both the second-price and first-price auctions, as all bidders have an equal probability of winning. This consistency underpins the applicability of Proposition 2 to both standard first-price and second-price auctions.

Corollary 2. *Under the assumptions made in the proposed model, a critical default probability, p_c, always exists such that $\pi(\mathcal{A}^{SP}) < 0$ for all default probability $p > p_c$, while $\pi(\mathcal{A}^{P}) \geq 0$ for all $p \in [0, 1]$.*

This implies that there is no reason for the rational seller to conduct $\mathcal{A}^{SP}$ if $p > p_c$ but the revenue from $\mathcal{A}^{P}$ is always non-negative irrespective of the the value of p. The divergence in outcomes between auctions that incorporate bidder default risk information and those that do not highlights the importance of risk-informed mechanism design. In a standard auction mechanism, such as the second-price auction ($\mathcal{A}^{SP}$), the seller's expected revenue can turn negative when the probability of bidder default (p) exceeds a critical threshold (p_c). This occurs because the mechanism fails to account for the likelihood of unrealized payments due to defaulting bidders. When defaults are frequent, the seller faces administrative costs, delays in asset allocation, and potential legal disputes, which collectively erode the net revenue to the extent that it becomes negative. For instance, in public-private partnership (PPP) infrastructure projects, bidders often default on financial commitments due to macroeconomic volatility or internal financial instability [Menezes and Ryan, 2013]. A standard auction design in such cases could lead to seller's negative payoff resulting in significant project delays, increased costs, and a failure to meet public service goals. Another practical example can be observed in real estate auctions where high-value properties attract speculative bidders with weak financial backing [Mäenpää, 2024]. In the absence of risk-informed mechanisms, the seller may risk negative payoff due to forfeiting the winning bid amount, incurring holding costs, and reinitiating the auction process.

Conversely, the proposed auction mechanism, which explicitly incorporates bidder default risk into its design, ensures positive expected revenues even when p is significantly high. By integrating risk information, the mechanism can implement features such as differentiated reserve prices for reliable and unreliable bidders, prioritization of reliable bidders, or dynamic bid adjustments based on default probabilities. These adjustments mitigate the revenue impact of defaults, aligning the mechanism with economic principles that address adverse selection and moral hazard. For example, in government auctions of spectrum licenses, the incorporation of reliability metrics, such as financial guarantees or track records, allows for a fair allocation while minimizing the risk of defaults that could disrupt public telecommunications projects. Similarly, in procurement auctions for defense contracts, integrating bidder reliability ensures that contracts are awarded to firms capable of fulfilling obligations, reducing potential fiscal losses for the government. The practical implications of the proposed mechanism are profound. It ensures fiscal prudence by safeguarding the seller's revenue streams, enhances efficiency by allocating assets to bidders most likely to perform, and builds credibility in the market by addressing default risks proactively. By doing so, it provides a sustainable auction framework that can adapt to environments with high uncertainty or default risks, protecting public and private interests alike.

4.1 A Special Case of Uniform Distribution

In this section, we examine a specific scenario where bidders' private values are sampled from a uniform distribution. This approach aligns with the consideration and assumption of [Vickrey, 1961, Menicucci, 2009, Kaplan and Zamir, 2007]. When the private values are uniformly distributed over the interval $[0, 1]$, the following *Lemma* and *Propositions* hold:

Lemma 3. *If the bidders' private values are drawn from a uniform distribution $F(x) = x$ with support on $[0, 1]$, the expected value of b_h (as defined in step 3 of mechanism 3.5) is given by:*

$$E[b_h] = \frac{n-2}{n} + \frac{1}{n\,2^{n-1}}$$

Proposition 3. *If the bidders' private values are drawn from a uniform distribution $F(x) = x$ with support on $[0, 1]$, then for any $n \geq 3$ and $C > \frac{1}{n+1}$ the following results will hold:*

1. *There always exists a threshold default probability p^* such that for all $p \geq p^*$, it is ensured that $\pi(\mathcal{A}^P) \geq \pi(\mathcal{A}^{SP})$.*

2. *The value of p^* is given by:*

$$p^* = \frac{A_1(1+C) - B_1 + \sqrt{[A_1(1+C) - B_1]^2 - 4A_1C(A_1 - B_2)}}{2A_1C}$$

where

$$A_1 = \frac{n-2}{n} + \frac{1}{n2^{n-1}}, \qquad B_1 = \frac{(n-1)^2}{n(n+1)} + \frac{C(n-1)}{n}$$

$$B_2 = \frac{n-1}{n(n+1)} + \frac{(n-1)^2}{n(n+1)} + \frac{C}{n} + \frac{C(n-1)}{n} - C$$

Refer to *Table* 1 for the values of p^* corresponding to various values of n with $C = 0.5$, where bidders' private valuations are drawn from the uniform distribution as described earlier. Observed that p^* decreases monotonically as n increases.

	n=3	n=4	n=5	n=6	n=7	n=8	n=9
p*	0.57	0.53	0.4	0.32	0.28	0.25	0.21

Table 1: Values of p^* for different values of n and $C = 0.5$

Corollary 3. *Proposition 3 holds for any value of C satisfying* $0 < C < 1$ *as* $n \to \infty$.

Consider another situation when the bidders draw their private values from a uniform distribution with support in $[0, 1]$ i.e. $F(x) = x$ and $f(x) = 1$. Then we compute the $\mathcal{R}(\mathcal{A}^P)$ and $\mathcal{R}(\mathcal{A}^{SP})$ as follows:

$$\mathcal{R}(\mathcal{A}^{SP}) = \frac{1 + (1-p)(n-1)}{n}$$

$$\begin{aligned}\mathcal{R}(\mathcal{A}^P) &= (1-p)F[(1-p)E[b_h]] + 1 - F[(1-p)E[b_h]]\\ &= 1 - pF[(1-p)E[b_h]]\\ &= 1 - p(1-p)E[b_h] \quad (\text{as } F(x) = x)\\ &= 1 - p(1-p)\left(\frac{n-2}{n} + \frac{1}{n\,2^{n-1}}\right)\end{aligned}$$

Here, b_h denotes the highest bid placed during step 3 of mechanism 3.5. Refer to *Figure* 1, which illustrates the reliability trends of $\mathcal{A}^{SP}$ and $\mathcal{A}^P$ with respect to p. The reliability of $\mathcal{A}^{SP}$ decreases monotonically as p increases. In contrast, the reliability of $\mathcal{A}^P$ initially declines with p but subsequently begins to rise after reaching a local minimum. This behavior arises because, beyond a certain threshold of p, the reliable bidder—characterized by a default probability of 0—is more likely to win, thereby enhancing the reliability of $\mathcal{A}^P$ as p surpasses the threshold. *Figure* 2 exhibits a comparison of the slope of $\mathcal{R}(\mathcal{A}) - p$ curve at different values of p. Refer to

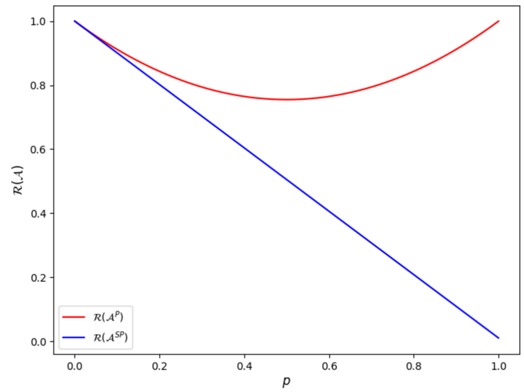

Figure 1: Reliability Comparison

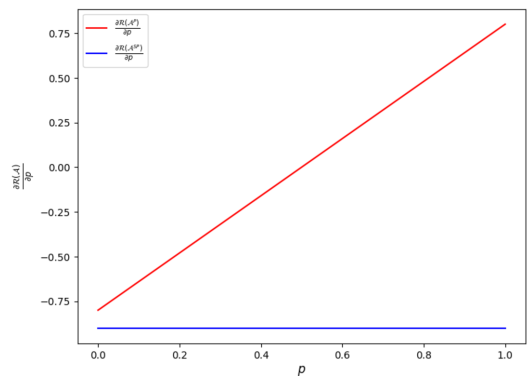

Figure 2: Slope comparison for Reliability-default probability graph

Figure 3, which demonstrates that the expected revenue of $\mathcal{A}^P$ exceeds that of $\mathcal{A}^{SP}$ at $p \approx 0.4$ when $n = 5$ and $C = 0.5$. Additionally, we observed that the revenue of $\mathcal{A}^{SP}$ becomes negative once p exceeds approximately 0.72, rendering the second-price auction entirely infeasible for the auctioneer beyond this threshold. *Figure* 4 illustrates the variation in p^* with C and $n \geq 10$, the number of unreliable bidders. Figure 5 illustrates *Corollary* 2 for the case where bidders independently draw their private values from a uniform distribution over the interval $[0, 1]$. The figure demonstrates that the critical probability of (as referenced in *Corollary* 2) of $\mathcal{A}^{SP}$ decreases with the increase in C and n. Notably, the influence of C on the critical probability is more substantial compared to n, particularly when n becomes significantly large.

4.2 Discussion on the Value of Risk Information

When the default probability of unreliable bidders is substantially high, the proposed auction mechanism demonstrates superior efficacy, both in terms of expected revenue and reliability,

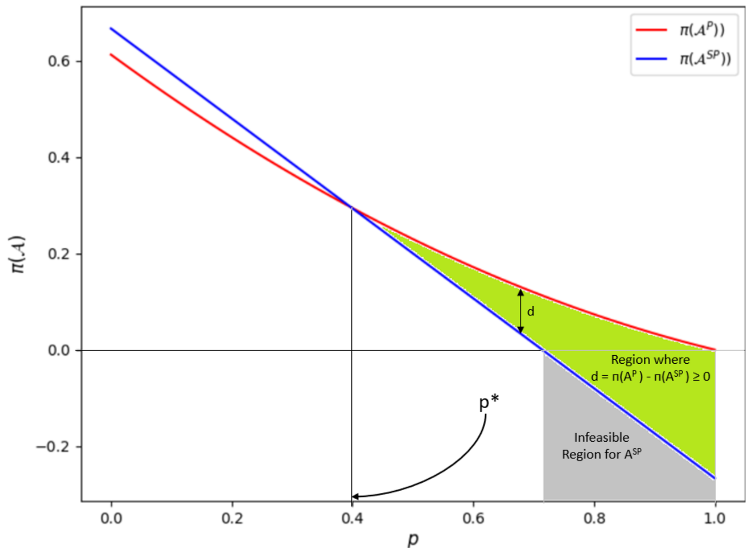

Figure 3: Revenue Comparison

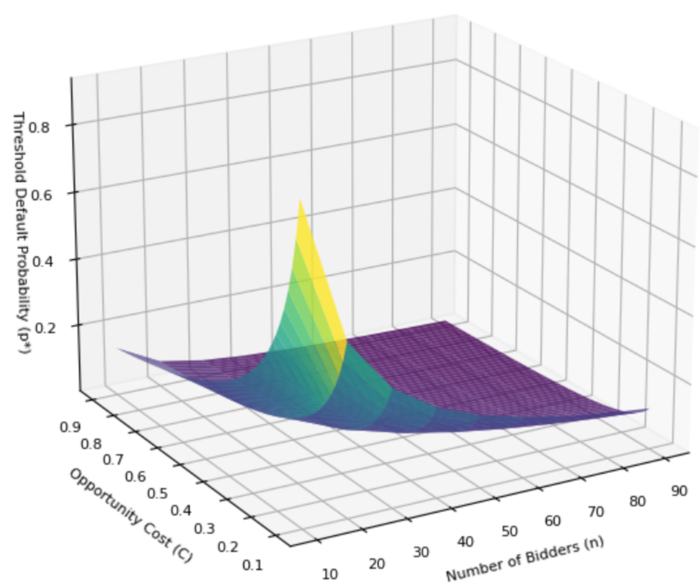

Figure 4: Variation of p^* with n and C

compared to standard second-price or first-price auctions that lack bidder reliability information. This advantage arises because the proposed mechanism effectively internalizes the economic cost of defaults by prioritizing reliable bidders or incorporating risk-based adjustments in bid evaluation. In traditional auction formats, high default probabilities among bidders can

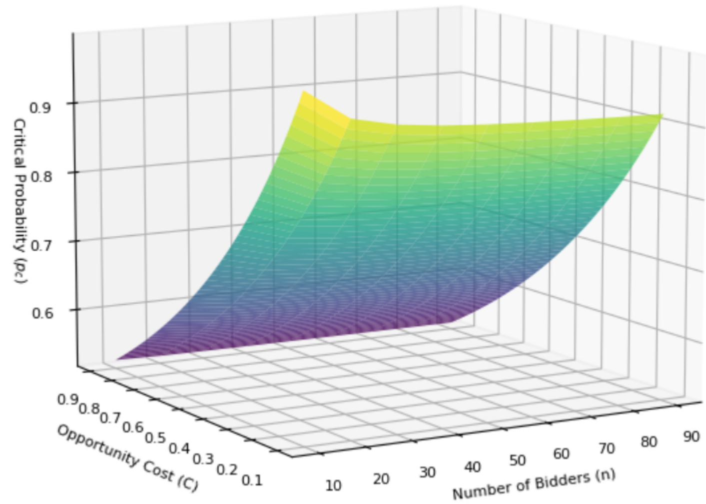

Figure 5: Illustration of *Corollary* 2 for the case where bidders' private values are independently drawn from a uniform distribution over $[0, 1]$. The critical probability of $\mathcal{A}^{SP}$ decreases with increasing values of C and n.

lead to inefficient allocations, reduced revenue certainty, and increased transaction costs for the seller due to post-auction complications, such as re-engagement with secondary bidders. The proposed mechanism mitigates these inefficiencies by leveraging the seller's knowledge of bidder reliability to design allocation rules that minimize default risks and stabilize revenue. Consequently, it aligns with optimal auction theory by improving allocative efficiency and reducing the economic inefficiencies associated with bidder defaults, thereby outperforming conventional auction formats in high-risk environments.

5 Policy Implication

The implementation of a reliable auction mechanism that minimizes the likelihood of bidder defaults while maintaining expected profit holds profound policy implications, particularly in contexts where defaults impose substantial economic and fiscal costs on governments and private entities. Bidder defaults often create a ripple effect of inefficiencies, including delays in project initiation, cost escalations, and administrative burdens associated with re-tendering or renegotiating contracts. For governments, these inefficiencies can translate into increased fiscal expenditures, as they are often required to allocate additional public resources to mitigate the consequences of defaults. Such expenditures may strain public budgets, divert funds from critical development initiatives, and undermine fiscal discipline, ultimately affecting taxpayers who bear the burden of these inefficiencies. In infrastructure projects, for instance, delays caused by defaulting bidders can disrupt public service delivery, resulting in economic inefficiencies and social dissatisfaction.

For private firms, bidder defaults negatively impact shareholder value by destabilizing revenue streams and introducing significant operational risks. Defaults may force firms to engage in costly legal disputes, re-negotiations, or additional due diligence, leading to unanticipated expenditure. This can erode investor confidence, lower stock prices, and diminish the firm's ability to attract capital. The uncertainty surrounding defaults also creates inefficiencies in resource allocation, as firms must reserve contingency funds to absorb potential losses instead of channeling these funds into productive ventures. Reliable auctions mitigate these issues by incentivizing bidder credibility through mechanisms that prioritize reliability over speculative participation, reducing the frequency and impact of defaults.

From a broader economic perspective, reliable auctions enhance market efficiency by ensuring that goods and services are allocated to participants most capable of fulfilling their obligations. This improves resource allocation and project implementation timelines, fostering a more predictable and stable economic environment. For governments, the reduction in fiscal burden enables better budgetary planning and resource allocation, allowing for the redirection of savings toward public goods and welfare programs. In addition, minimizing defaults can bolster public trust in procurement processes, encouraging greater participation from credible bidders and enhancing competition, which can lead to better outcomes in terms of quality and cost.

For private firms, reliable auction mechanisms translate into stronger financial performance by stabilizing cash flows and reducing the risk premiums associated with uncertain transactions. Firms are better positioned to optimize their operational strategies and focus on long-term value creation, enhancing their competitiveness in the market. Furthermore, reducing the prevalence of bidder defaults aligns with broader economic objectives, such as fostering trust and stability in financial and commercial markets, which are critical for sustained economic growth and development. Policymakers and corporate leaders should, therefore, recognize the strategic importance of integrating reliability criteria into auction designs to balance profit objectives with systemic stability and social welfare.

6 Future Research Direction & Conclusion

The interplay between exogenous factors limiting bidders' financial commitments and the seller's incomplete information introduces significant complexities into auction markets, necessitating a reexamination of traditional auction models. When bidders are constrained by external circumstances—such as liquidity limitations, dependency on external financing, or regulatory restrictions—their ability to honor commitments is inherently uncertain. Simultaneously, when sellers lack full visibility into these constraints, adverse selection and moral hazard issues emerge, compromising auction efficiency and outcomes. These challenges underscore the critical need for auction mechanisms that can simultaneously address bidder heterogeneity and seller uncertainty. From an economic perspective, the presence of exogenous financial constraints distorts bidders' strategies, leading to a potential misalignment between bids and actual valuations. Bidders may understate their willingness to pay due to liquidity concerns or overstate it in hopes of securing post-auction financing, resulting in allocative inefficiencies. Sellers, facing incomplete information, are often unable to differentiate between financially viable and constrained bidders, increasing the likelihood of awarding goods or services to bidders who ultimately default. This mismatch can lead to suboptimal revenue outcomes, reputational risks, and inefficiencies in resource allocation. Furthermore, the introduction of exogenous constraints alters the risk landscape, shifting a portion of the auction's inherent risk from bidders to the seller.

Future research in this domain should prioritize the development of advanced auction mechanisms that incorporate bidder-specific constraints and address the seller's informational limitations. One promising avenue involves integrating screening mechanisms to elicit truthful disclosure of financial constraints. This could include the design of prequalification criteria, performance bonds, or the incorporation of reputation systems that penalize defaulting bidders in repeated auction environments. Additionally, research could explore dynamic auction models where bids and commitments are adjusted iteratively as new information becomes available, allowing for a more accurate reflection of bidders' financial capacities. Mechanism design must also consider risk-sharing strategies to mitigate the potential consequences of bidder defaults. Auctions with staggered payments or contingent pricing mechanisms, where a portion of the payment depends on post-auction performance, could align the interests of bidders and sellers more effectively. Furthermore, introducing hybrid auction formats, which combine elements of traditional auctions with negotiation phases for payment terms, could provide a robust framework for addressing financial constraints.

Empirical analysis represents another critical frontier. By studying real-world auction failures due to bidder defaults or renegotiations, researchers can gain valuable insights into the practical challenges of auctions under uncertainty. Sector-specific studies, such as in real estate, spectrum allocation, and procurement auctions, where financial constraints are prevalent, can offer lessons on the design of more resilient auction frameworks. Behavioral economics could further enrich this analysis by examining how bidders' cognitive biases, such as optimism or overconfidence, interact with financial constraints to influence bidding behavior.

In conclusion, addressing the challenges posed by exogenous bidder constraints and incomplete seller information requires a multidisciplinary approach. Combining theoretical modeling, empirical validation, and behavioral insights, future research has the potential to enhance the efficiency, fairness, and robustness of auction mechanisms. By doing so, the field can contribute to bridging the gap between economic theory and the practical realities of auction markets, fostering better outcomes for both bidders and sellers in diverse contexts.

References

Mohammad Akbarpour and Shengwu Li. Credible auctions: A trilemma. *Econometrica*, 88 (2):425–467, 2020.

Malin Arve and David Martimort. Auctioning long-term projects under financial constraints. *Review of Economic Studies*, page rdae106, 2024.

Sulin Ba and Paul A Pavlou. Evidence of the effect of trust building technology in electronic markets: Price premiums and buyer behavior. *MIS quarterly*, pages 243–268, 2002.

Aniruddha Bagchi, Brett Katzman, and Timothy Mathews. Second chance offers in auctions. *Journal of Economics*, 112, 01 2013. doi: 10.1007/s00712-012-0330-4.

Oleksii Birulin. Optimality of simple procurement auctions. *International Journal of Industrial Organization*, 70:102610, 2020.

Simon Board. Bidding into the red: A model of post-auction bankruptcy. *Journal of Finance*, 62:2695–2723, 02 2007. doi: 10.1111/j.1540-6261.2007.01290.x.

Gary E Bolton, Elena Katok, and Axel Ockenfels. *How Effective are Online Reputation Mechanisms?: An Experimental Investigation*. Max Planck Inst. for Research into Economic Systems, Strategic Interaction Group, 2002.

Norris Bruce, Ernan Haruvy, and Ram Rao. Seller rating, price, and default in online auctions. *Journal of Interactive marketing*, 18(4):37–50, 2004.

Luis Cabral and Ali Hortacsu. The dynamics of seller reputation: Evidence from ebay. *The journal of industrial economics*, 58(1):54–78, 2010.

Yeon-Koo Che and Ian Gale. Standard auctions with financially constrained bidders. *The Review of Economic Studies*, 65(1):1–21, 1998. URL `https://EconPapers.repec.org/RePEc:oup:restud:v:65:y:1998:i:1:p:1-21`.

Shu-Chen Kao ChienHsing Wu and ChengHsun Ho. Determinants of nonpaying bid behavior in online auction platforms. *Journal of Internet Commerce*, 21(1):51–78, 2022. doi: 10.1080/15332861.2021.1889817. URL `https://doi.org/10.1080/15332861.2021.1889817`.

Ottorino Chillemi and Stefano Galavotti. Optimal procurement auctions with audit. *Economic Theory*, pages 1–62, 2024.

Domenico Colucci, Nicola Doni, and Vincenzo Valori. Preferential treatment in procurement auctions through information revelation. *Economics Letters*, 117(3):883–886, 2012. ISSN 0165-1765. doi: https://doi.org/10.1016/j.econlet.2012.07.001. URL `https://www.sciencedirect.com/science/article/pii/S0165176512003862`.

Chrysanthos Dellarocas and Charles A Wood. The sound of silence in online feedback: Estimating trading risks in the presence of reporting bias. *Management science*, 54(3):460–476, 2008.

Peter M. DeMarzo, Ilan Kremer, and Andrzej Skrzypacz. Bidding with securities: Auctions and security design. *American Economic Review*, 95(4):936–959, 2005.

Alexander K. Koch Dirk Engelmann, Jeff Frank and Marieta Valente. Second-chance offers and buyer reputation systems: theory and evidence on auctions with default. *The RAND Journal of Economics*, 2023.

Andreas R Engel and Achim Wambach. Risk management in procurement auctions. *Quaderni Consip*, 10, 2005.

Paul Ferrell. Dead weight loss, windfall profit, and the solution to the helium market. *Department of Economics Stanford University*, 2014.

Dawn G Gregg and Judy E Scott. The role of reputation systems in reducing on-line auction fraud. *International Journal of Electronic Commerce*, 10(3):95–120, 2006.

Vitali Gretschko and Helene Mass. Worst-case equilibria in first-price auctions. *Theoretical Economics*, 19(1):61–93, 2024.

Manika Gupta. Bidders withdraw from highway projects. https://timesofindia.indiatimes.com/business/india-business/bidders-withdraw-from-highway-projects/articleshow/3720250.cms, 2008.

Ronald M. Harstad and Michael H. Rothkopf. Withdrawable bids as winner's curse insurance. *Operations Research*, 43(6):983–994, 1995. ISSN 0030364X, 15265463. URL http://www.jstor.org/stable/171639.

Birgit Heydenreich, Rudolf Müller, Marc Uetz, and Rakesh V Vohra. Characterization of revenue equivalence. *Econometrica*, 77(1):307–316, 2009.

Alan Holland and Barry O'Sullivan. Truthful risk-managed combinatorial auctions. In *IJCAI*, pages 1315–1320. Citeseer, 2007.

Shahid Hussain, Mohsin Shahzad, Andrea Appolloni, and Wang Xuetong. The impact of public infrastructure project delays on sustainable community development. *Environmental Science and Pollution Research*, 30, 01 2023. doi: 10.1007/s11356-022-24739-2.

Rajkamal Iyer and Antoinette Schoar. Contracting when enforcement is weak: evidence from an audit study. *Review of Finance*, 28(5):1513–1536, 2024.

Todd Kaplan and Shmuel Zamir. Asymmetric first-price auctions with uniform distributions: Analytic solutions to the general case. *Economic Theory*, 50, 12 2007. doi: 10.2139/ssrn.930518.

Judith Kas, Rense Corten, and Arnout van de Rijt. Trust, reputation, and the value of promises in online auctions of used goods. *Rationality and Society*, 35(4):387–419, 2023. doi: 10.1177/10434631231170342.

Paul Klemperer. *Auctions: Theory and Practice*. Princeton University Press, 2004. URL http://www.jstor.org/stable/j.ctv3hh4pm.

Vijay Krishna. *Auction Theory*. Elsevier Inc., United States, 2003. ISBN 9780124262973. doi: 10.1016/B978-0-12-426297-3.X5026-7.

Jennifer Lamping. The value of information in auctions with default risk. *ERN: Search*, 2007. URL https://api.semanticscholar.org/CorpusID:49581439.

Joosung Lee and Daniel Z. Li. Participation and welfare in auctions with default. *Economics Letters*, 183:108620, 2019. ISSN 0165-1765. doi: https://doi.org/10.1016/j.econlet.2019.108620. URL https://www.sciencedirect.com/science/article/pii/S0165176519303052.

Jeff Livingston. How valuable is a good reputation? a sample selection model of internet auctions. *The Review of Economics and Statistics*, 87:453–465, 02 2005. doi: 10.1162/0034653054638391.

Panos L. Lorentziadis. Bidding under auctioneer default risk. *Omega*, 49:123–133, 2014. ISSN 0305-0483. doi: https://doi.org/10.1016/j.omega.2014.05.005. URL https://www.sciencedirect.com/science/article/pii/S030504831400067X.

Jingfeng Lu. Auction design with opportunity cost. *Economic Theory*, 38:73–103, 2009.

David Lucking-Reiley, Doug Bryan, Naghi Prasad, and Daniel Reeves. Pennies from ebay: The determinants of price in online auctions. *The journal of industrial economics*, 55(2): 223–233, 2007.

Mikhail I Melnik and James Alm. Does a seller's ecommerce reputation matter? evidence from ebay auctions. *The journal of industrial economics*, 50(3):337–349, 2002.

Flavio Menezes and Matthew Ryan. Default and renegotiation in ppp auctions. *Journal of Public Economic Theory*, 17, 12 2013. doi: 10.1111/jpet.12102.

Domenico Menicucci. Harmful competition in all-pay auctions. *Mathematical Social Sciences*, 58:110–120, 07 2009. doi: 10.1016/j.mathsocsci.2009.01.003.

Yizhaq Minchuk and Aner Sela. Prebidding first-price auctions with and without head starts. *Mathematical Social Sciences*, 91:51–55, 2018. ISSN 0165-4896. doi: https://doi.org/10.1016/j.mathsocsci.2017.07.001. URL https://www.sciencedirect.com/science/article/pii/S0165489617300380.

R Charles Moyer and Robert Chatfield. Market power and systematic risk. *Journal of Economics and Business*, 35(1):123–130, 1983.

Eero Mäenpää. Auctions with outside offers. 01 2024. doi: 10.2139/ssrn.5071915.

Pacific Debt Relief. Understanding the debt collection and legal process, 2023. URL https://www.pacificdebt.com/understanding-the-debt-collection-and-legal-process.

Marco Pagnozzi and Krista J. Saral. Auctions with limited liability through default or resale. *Journal of Economic Behavior Organization*, 159:51–74, 2019. ISSN 0167-2681. doi: https://doi.org/10.1016/j.jebo.2019.01.011. URL https://www.sciencedirect.com/science/article/pii/S016726811930023X.

Salima Y Paul, S Susela Devi, and Chee Ghee Teh. Impact of late payment on firms' profitability: Empirical evidence from malaysia. *Pacific-Basin Finance Journal*, 20(5):777–792, 2012.

Matthew R Roelofs. Common value auctions with default: An experimental approach. *Experimental Economics*, 5:233–252, 2002.

Alejandro Toriello Sebastian Perez-Salazar, Mohit Singh. Robust online selection with uncertain offer acceptance. *Mathematics of Operations Research*, 2024. doi: https://doi.org/10.1287/moor.2023.0210.

Susan Sunila Sharma, Paresh Kumar Narayan, and Xinwei Zheng. An analysis of firm and market volatility. *Economic Systems*, 38(2):205–220, 2014. ISSN 0939-3625. doi: https://doi.org/10.1016/j.ecosys.2013.12.003. URL `https://www.sciencedirect.com/science/article/pii/S093936251400034X`. Symposium: Performance of Financial Markets.

Giancarlo Spagnolo, Gian Luigi Albano, and Milo Bianchi. Bid avarage methods in procurement. 2006.

S Tadelis. Reputation and feedback systems in online platform markets. *Journal of Political Economy*, 92:854–882, 2002.

Steven Tadelis. Reputation and feedback systems in online platform markets. *Annual Review of Economics, Annual Reviews*, 8:321–340, 10 2016.

Deniz Ucbasaran, Dean A. Shepherd, Andy Lockett, and S. John Lyon. Life after business failure: The process and consequences of business failure for entrepreneurs. *Journal of Management*, 39(1):163–202, 2013. doi: 10.1177/0149206312457823.

William S. Vickrey. Counterspeculation, auctions, and competitive sealed tenders. *Journal of Finance*, 16:8–37, 1961. URL `https://api.semanticscholar.org/CorpusID:153426451`.

Cindy Vojtech, Benjamin Kay, and John Driscoll. The real consequences of bank mortgage lending standards. *SSRN Electronic Journal*, 01 2016. doi: 10.2139/ssrn.2780468.

Matthew Walker et al. *Trust and Trustworthiness in Imbalanced Markets*. PhD thesis, Durham University, 2021.

Appendix A: Proofs

Proof of proposition 1.

Proof. According to the mechanism 3.5 of $\mathcal{A}^P$, one of the unreliable bidders wins the auction when the bid(b_h) placed by him is greater than the bids placed by other unreliable bidders in the *step* 3 of mechanism 3.5 and the private value of the reliable bidder is lower than $(1-p)b_h$. On the other hand, the reliable bidder wins the auction when his private value is higher than $(1-p)b_h$. Therefore, we can write the reliability (Ref. *definition* 1) of $\mathcal{A}^P$ as follows:

$$\begin{aligned}\mathcal{R}(\mathcal{A}^P) &= (1-p)F[(1-p)b_h] + (1-F[(1-p)b_h]) \\ &= 1-pF[(1-p)b_h]\end{aligned} \tag{1}$$

On the other hand, in $\mathcal{A}^{SP}$ all the bidders are equally likely to win the auction as there is no differentiation based on the default probability of the bidders. Therefore,

$$\mathcal{R}(\mathcal{A}^{SP}) = \frac{(n-1)(1-p)}{n} + \frac{1}{n} = \frac{1+(n-1)(1-p)}{n} \tag{2}$$

Observe that $\mathcal{R}(\mathcal{A}^{SP})$ monotonically decreases with p with slope $\frac{n-1}{n}$ as $\frac{\partial \mathcal{R}(\mathcal{A}^{SP})}{\partial p} = -\frac{n-1}{n} < 0$.

$$\mathcal{R}(\mathcal{A}^{P})|_{p=0} = 1 \quad \text{and} \quad \mathcal{R}(\mathcal{A}^{SP})|_{p=0} = 1 \implies \mathcal{R}(\mathcal{A}^{P})|_{p=0} = \mathcal{R}(\mathcal{A}^{SP})|_{p=0}$$

Conversely,

$$\mathcal{R}(\mathcal{A}^{P})|_{p=1} = 1 \quad \text{and} \quad \mathcal{R}(\mathcal{A}^{SP})|_{p=1} = \frac{1}{n} \implies \mathcal{R}(\mathcal{A}^{P})|_{p=1} > \mathcal{R}(\mathcal{A}^{SP})|_{p=1}$$

This implies that the $\mathcal{R}(\mathcal{A}^{P}) - p$ curve and the $\mathcal{R}(\mathcal{A}^{SP}) - p$ curve intersect at least once within the interval $[0,1]$. Let p^* denote the largest value of p at which these two curves intersect in $[0,1]$. Consequently, for all $p^* < p \leq 1$, it follows that $\mathcal{R}(\mathcal{A}^{P}) > \mathcal{R}(\mathcal{A}^{SP})$, given that $\mathcal{R}(\mathcal{A}^{P})\Big|_{p=1} > \mathcal{R}(\mathcal{A}^{SP})\Big|_{p=1}$ and both $\mathcal{R}(\mathcal{A}^{P}) - p$ and $\mathcal{R}(\mathcal{A}^{SP}) - p$ curves are continuous in $[0,1]$. This completes the proof.

Proof of Lemma 1

Proof. Lets assume the provisional winning bid at *step* 2 in mechanism 3.5 is b_w. Let us also designate highest and the second-highest private value among the $n-1$ unreliable bidders by v_1 and v_2. In equilibrium, b_w maximizes the winning bidder's pay-off at *step* 2. The winning bidder maximizes his pay-off π_b with respect to b_w as follows:

$$\pi_b = F^{n-2}(v_1)(v_1 - b_w)F(b_w) \tag{3}$$

The FOC of the above equation provides $b_w = v_1 - \frac{F(b_w)}{f(b_w)}$. Now, we have to derive the condition that guarantees that the value of b_w as derived from the FOC above indeed maximizes π_b i.e. $\frac{\partial^2 \pi_b}{\partial b_w^2} < 0$. We can derive this condition as follows:

$$\frac{\partial^2 \pi_b}{\partial b_w^2} = F^{N-2}(v_1)[(v_1 - b_w)f'(b_w) - 2f(b_w)] \tag{4}$$

Note that $0 \leq (v_1 - b_w) \leq v_1 \leq 1$ because of the individual rationality assumption for each bidder. Therefore, the set of regular distributions $f(x)$ which satisfy $2f(x) > f'(x)$ for all $x \in [0,1]$ always guarantees that $\frac{\partial^2 \pi_b}{\partial b_w^2} = F^{N-2}(v_1)[(v_1 - b_w)f'(b_w) - 2f(b_w)] < 0$ i.e. π_b maximizes at $b_w = v_1 - \frac{F(b_w)}{f(b_w)} = g(v_1)$ if $2f(x) > f'(x)$ for all $x \in [0,1]$. Therefore, $g(v_1) < v_1$ and it is continuous and differentiable as $f(x) \neq 0$ for all $x \in [0,1]$.

In this situation, following two scenarios can arise: $b_w > v_2$ or $b_w < v_2$. There are $(n-1)(n-2)$ possible pairs of highest valuation bidder and second-highest valuation bidder among $n-1$ unreliable bidders given that all the bidders are equally likely to be highest or second highest valuation bidder.

Case I: $b_w > v_2$

If $b_w > v_2$, then in the bid adjustment process of *step* 3 of mechanism 3.5, the bidder with second-highest value can't adjust its bid and that results in $b_h = b_w$. Therefore, the expected value of b_h in this scenario is given by:

$$E_1[b_h] = (n-1)(n-2)\int_0^1 \int_0^{b_w} b_w F^{n-3}(v_2) f(v_1) f(v_2) dv_1 dv_2$$

Case II: $b_w < v_2$

If $b_w < v_2$, then the bid adjustment process of *step* 3 of mechanism 3.5 continues until the adjusted bid from the bidder with highest valuation exceeds v_2. This results in $b_h = v_2$. Therefore, the expected value of b_h in this scenario given by:

$$E_2[b_h] = (n-1)(n-2)\int_0^1 \int_{b_w}^{v_1} v_2 F^{n-3}(v_2) f(v_1) f(v_2)\, dv_1 dv_2$$

Therefore, the total expected value of b_h is as follows:

$$\begin{aligned} E[b_h] &= E_1[b_h] + E_2[b_h] \\ &= (n-1)(n-2)\Big[\int_0^1 \int_0^{b_w} b_w F^{n-3}(v_2) f(v_1) f(v_2)\, dv_1 dv_2 \\ &\qquad + \int_0^1 \int_{b_w}^{v_1} v_2 F^{n-3}(v_2) f(v_1) f(v_2)\, dv_1 dv_2\Big] \\ &= (n-1)(n-2)\Big[\int_0^1 \int_0^{g(v_1)} g(v_1) F^{n-3}(v_2) f(v_1) f(v_2)\, dv_1 dv_2 \\ &\qquad + \int_0^1 \int_{g(v_1)}^{v_1} v_2 F^{n-3}(v_2) f(v_1) f(v_2)\, dv_1 dv_2\Big] \end{aligned} \tag{5}$$

Proof of Lemma 2

Proof. Let v_2 denote the second-highest value from n independent draws from a regular distribution. We aim to determine the probability that $v_2 \in [1-\epsilon, 1]$, where $\epsilon > 0$ is small. The probability density function (PDF) of the second-highest value v_2 is:

$$f_2(x) = n(n-1)F^{n-2}(x)[1-F(x)]f(x),$$

On integrating $f_2(x)$ over $[1-\epsilon, 1]$:

$$\begin{aligned}
P(v_2 \in [1-\epsilon, 1]) &= \int_{1-\epsilon}^{1} f_2(x)\,dx \\
&= n(n-1)\int_{1-\epsilon}^{1} [F(x)]^{n-2}[1-F(x)]f(x)dx \\
&= n(n-1)\int_{1-\epsilon}^{1} [F(x)]^{n-2}[1-F(x)]dF(x) \\
&= n(n-1)\int_{1-\epsilon}^{1} [F^{n-2}(x) - F^{n-1}(x)]dF(x) \\
&= n(n-1)\left[\frac{1-F^{n-1}(1-\epsilon)}{n-1} - \frac{1-F^{n}(1-\epsilon)}{n}\right] \\
&= \left[1 - nF^{n-1}(1-\epsilon) + (n-1)F^{n}(1-\epsilon)\right]
\end{aligned} \tag{6}$$

As $f(x)$ is a regular distribution, we know that:

$$\frac{1-F(x)}{f(x)} = h(x) \text{ where } h(x) \text{ is a non-increasing function of } x$$

Therefore, we can derive the nature of $F(x)$ as follows:

$$\begin{aligned}
& \frac{1-F(x)}{f(x)} = h(x) \\
\implies & \frac{1}{h(x)} = \frac{f(x)}{1-F(x)} \\
\implies & \frac{dx}{h(x)} = \frac{f(x)dx}{1-F(x)} \\
\implies & -\frac{dx}{h(x)} = \frac{d[1-F(x)]}{1-F(x)} \\
\implies & -\int \frac{dx}{h(x)} = \int \frac{d[1-F(x)]}{1-F(x)} \\
\implies & -\int \frac{dx}{h(x)} = ln\left[\frac{1}{k}(1-F(x)\right] \text{ (k is a constant)} \\
\implies & 1-F(x) = k\, e^{-\int \frac{1}{h(x)}dx} \\
\implies & F(x) = 1 - k\, e^{-\int \frac{dx}{h(x)}}
\end{aligned} \tag{7}$$

This implies that $1-F(x)$ decays exponentially, causing $F(x)$ to grow rapidly for small values of x but its growth slows down significantly as x approaches 1. Consequently, the following approximation holds when ϵ is sufficiently small:

$$F^{n-1}(1-\epsilon) \approx F^{n}(1-\epsilon) \text{ (as } F(1) = 1)$$

Substituting this into equation 6 we get,

$$P(v_2 \in [1-\epsilon, 1]) \approx 1 - F^{n-1}(1-\epsilon) \approx 1 \text{ (as } F(1-\epsilon) < 1 \text{ and } n \to \infty)$$

This completes the proof.

Proof of Lemma 3

Proof. We will first derive $b_w = g(v)$ (Refer to *Lemma* 1). The expected pay-off of the the highest valuation bidder from $n-1$ unreliable bidder as follows:

$$\pi = (1-p)F^{n-2}(v_1)(v_1 - b_w)F(b_w) = (1-p)v_1^{n-2}(v_1 - b_w)b_w$$

Here, v_1 represents the private value of the highest valuation bidder among $n-1$ unreliable bidder, b_w is the highest bid placed in *step* 2 of mechanism 3.5 and p is the probability to default of the unreliable bidders. We can derive the optimal value of b_w by the FOC of the above equation:

$$\frac{\partial \pi}{\partial b_w} = (1-p)v_1^{n-2}(-b_w + v_1 - b_w) = 0 \implies b_w = \frac{v_1}{2}$$

$b_w = \frac{v_1}{2}$ maximizes π because $b_w < \frac{v_1}{2}$ results in $\frac{\partial \pi}{\partial b_w} > 0$ and $b_w > \frac{v_1}{2}$ results in $\frac{\partial \pi}{\partial b_w} < 0$. Therefore, according to the proof of *Lemma* 1, we have

$$\begin{aligned}
E_1[b_h] &= (n-1)(n-2)\int_0^1 \int_0^{b_w} b_w F^{n-3}(v_2) f(v_1) f(v_2) dv_1 dv_2 \\
&= (n-1)(n-2)\int_0^1 \int_0^{\frac{v_1}{2}} \frac{v_1}{2}\, v_2^{n-3}\, dv_1 dv_2 \\
&= (n-1)(n-2)\int_0^1 \frac{v_1}{2} \int_0^{\frac{v_1}{2}} v_2^{n-3} dv_2 dv_1 \\
&= (n-1)\int_0^1 \frac{v_1}{2}.\left(\frac{v_1}{2}\right)^{n-2} dv_1 \\
&= (n-1)\int_0^1 \left(\frac{v_1}{2}\right)^{n-1} dv_1 \\
&= \frac{n-1}{n\, 2^{n-1}}
\end{aligned} \tag{8}$$

Similarly,

$$\begin{aligned}
E_2[b_h] &= (n-1)(n-2)\int_0^1 \int_{b_w}^{v_1} v_2 F^{n-3}(v_2) f(v_1) f(v_2)\, dv_1 dv_2 \\
&= (n-1)(n-2)\int_0^1 \int_{\frac{v_1}{2}}^{v_1} v_2\, v_2^{n-3} dv_1 dv_2 \\
&= (n-1)(n-2)\int_0^1 \int_{\frac{v_1}{2}}^{v_1} v_2^{n-2}\, dv_2\, dv_1 \\
&= (n-2)\int_0^1 \left[v_1^{n-1} - \left(\frac{v_1}{2}\right)^{n-1}\right] dv_1 \\
&= \frac{n-2}{n} - \frac{n-2}{n\, 2^{n-1}}
\end{aligned} \tag{9}$$

Therefore,

$$E[b_h] = E_1[b_h] + E_2[b_h] = \frac{n-1}{n\,2^{n-1}} + \frac{n-2}{n} - \frac{n-2}{n\,2^{n-1}} = \frac{n-2}{n} + \frac{1}{n\,2^{n-1}} \tag{10}$$

This completes the proof.

Proof of Proposition 2.

Proof.

1. Let us assume that v_2^{SP} is the second highest valuation among n bidders in a standard second price auction. As $n \to \infty$, we have $v_2^{SP} \approx 1$ (Ref. *Lemma* 2). On the other hand, the expected probability that the winning bidder will make the payment is given by

$$\mathcal{R}(\mathcal{A}^{SP}) = \frac{1 + (n-1)(1-p)}{n}$$

That implies the expected revenue from $\mathcal{A}^{SP}$ when $n \to \infty$ is as follows:

$$\begin{aligned} \pi(\mathcal{A}^{SP}) &\approx \frac{1 + (n-1)(1-p)}{n} - \left(1 - \frac{1 + (n-1)(1-p)}{n}\right)C \\ &= \left(\frac{1 + (n-1)(1-p)}{n}\right)(1 + C) - C \end{aligned} \tag{11}$$

$$\implies \frac{\partial \pi(\mathcal{A}^{SP})}{\partial p} \approx -\frac{n-1}{n}(1 + C) \approx -(1 + C) \text{ (as } n \to \infty)$$

Conversely, Let us assume that v_2^P is the second highest valuation among $n-1$ unreliable bidders in the auction $\mathcal{A}^P$ (Ref. mechanism 3.5). When $n \to \infty$, $v_2^P \approx 1$ (Ref. *Lemma* 2). As $b_h \geq v_2^P$ (Ref. mechanism 3.5), it implies that $b_h \approx 1$. Therefore, we can write the expected revenue of $\mathcal{A}^P$ as follows:

$$\pi(\mathcal{A}^P) = (1-p)b_h - pF[(1-p)b_h]C \approx (1-p) - pF[1-p]C \tag{12}$$

From *equation* 11 we get the following:

$$\pi(\mathcal{A}^{SP}) \approx \left(\frac{1 + (n-1)(1-p)}{n}\right)(1+C) - C \approx (1-p)(1+C) - C = (1-p) - pC \text{ (as } n \text{ is large)}$$

We can proceed with the final section of the proof as follows:

$$\begin{aligned} & pF(1-p)C < pC \\ \implies & -pF(1-p)C > -pC \\ \implies & (1-p) - pF(1-p)C > (1-p) - pC \\ \implies & \pi(\mathcal{A}^P) > \pi(\mathcal{A}^{SP}) \text{ (Ref. } \textit{equation} \text{ 11 and 12)} \end{aligned} \tag{13}$$

This completes the proof.

2. From equation 12
$$\frac{\partial \pi(\mathcal{A}^P)}{\partial p} \approx -1 - [F(1-p) - pf(1-p)]C$$

We can now establish the relationship between the sensitivity of $\pi(\mathcal{A}^P)$ and $\pi(\mathcal{A}^{SP})$ to the default probability p as follows:

$$\begin{aligned}
& F(1-p) < 1 \\
\implies & F(1-p) - pf(1-p) < 1 \\
\implies & [F(1-p) - pf(1-p)]C < C \\
\implies & -[F(1-p) - pf(1-p)]C > -C \\
\implies & -1 - [F(1-p) - pf(1-p)]C > -(1+C) \\
\implies & |1 + [F(1-p) - pf(1-p)]C| < |1+C| \\
\implies & \left|\frac{\partial \pi(\mathcal{A}^P)}{\partial p}\right| < \left|\frac{\partial \pi(\mathcal{A}^{SP})}{\partial p}\right|
\end{aligned} \tag{14}$$

This completes the proof.

Proof of Corollary 2.

Proof.

Suppose the highest bid in $\mathcal{A}^{SP}$ is R. $0 < R < 1$ as the bidders draw their valuations from distribution with support in $[0,1]$. We have from *equation* 1, the reliability of $\mathcal{A}^{SP}$ i.e. the expected probability that the winning bidder will not default is as follows:

$$\mathcal{R}(\mathcal{A}^{SP}) = \left(\frac{1 + (n-1)(1-p)}{n}\right)$$

Therefore,
$$\pi(\mathcal{A}^{SP}) = \mathcal{R}(\mathcal{A}^{SP})R - (1 - \mathcal{R}(\mathcal{A}^{SP}))C$$

Observe that $\pi(\mathcal{A}^{SP})$ decreases monotonically in p. Therefore, we need to prove that there exists a critical probability p_c such that $\pi(\mathcal{A}^{SP}) = 0$ at $p = p_c$. The monotonicity of $\pi(\mathcal{A}^{SP})$ ensures that $\pi(\mathcal{A}^{SP}) < 0$ for $p > p_c$. By solving the equation above, we get

$$0 < p_c = 1 - \frac{\frac{C}{R+C} - \frac{1}{n}}{1 - \frac{1}{n}} < 1$$

Similarly, from equation 12,

$$\pi(\mathcal{A}^P) = (1-p)b_h - p(1 - (1-p)b_h)C$$

Observe that $\pi(\mathcal{A}^P)$ decreases monotonically with p and $\pi(\mathcal{A}^P) = 0$ at $p = 1$. Therefore, $\pi(\mathcal{A}^P) \geq 0$ for all values of p.

Proof of Proposition 3.

Proof.

1. For $\mathcal{A}^{SP}$, the expected revenue to the bidder when all the bidders draw their private value from $U(0, 1)$ is as follows:

$$\pi(\mathcal{A}^{SP}) = \left(\frac{1 + (n-1)(1-p)}{n}\right)\frac{n-1}{n+1} - \left(1 - \frac{1 + (n-1)(1-p)}{n}\right)C$$

   Similarly, For $\mathcal{A}^P$, the expected revenue to the bidder when all the bidders draw their private value from $U(0, 1)$ is as follows:

$$\begin{aligned}\pi(\mathcal{A}^P) &= (1-p)b_h - F[(1-p)b_h]pC \\ &= (1-p)\left[\frac{n-2}{n} + \frac{1}{n\,2^{n-1}}\right] - p(1-p)\left[\frac{n-2}{n} + \frac{1}{n\,2^{n-1}}\right]C \text{ (Ref. } \textit{Lemma } 2\text{)}\end{aligned} \tag{15}$$

   Notice that $\pi(\mathcal{A}^P)\big|_{p=0} < \pi(\mathcal{A}^{SP})\big|_{p=0}$. Conversely, we have,

$$\pi(\mathcal{A}^P)\big|_{p=1} = 0 \quad \text{and} \quad \pi(\mathcal{A}^{SP})\big|_{p=1} = \frac{n-1}{n(n+1)} - \frac{(n-1)C}{n}$$

   Both $\pi(\mathcal{A}^P)$ and $\pi(\mathcal{A}^{SP})$ are monotonically decreasing in p. Therefore, the only condition for $\pi(\mathcal{A}^{SP})$ curve to intersect with $\pi(\mathcal{A}^P)$ curve is as follows:

$$\begin{aligned}&\pi(\mathcal{A}^{SP})_{p=1} < \pi(\mathcal{A}^P)_{p=1} \\ \implies &\frac{n-1}{n(n+1)} - \frac{(n-1)C}{n} < 0 \implies C > \frac{1}{n+1}\end{aligned} \tag{16}$$

   If $\pi(\mathcal{A}^P)$ curve intersects with $\pi(\mathcal{A}^{SP})$ curve and $\pi(\mathcal{A}^{SP})_{p=1} < \pi(\mathcal{A}^P)_{p=1}$, it ensures that there will always be a threshold probability p^* such that $\pi(\mathcal{A}^P) \geq \pi(\mathcal{A}^{SP})$ for all $p \geq p^*$. This completes the proof.

2. We can derive the expression of p^* by solving the equation $\pi(\mathcal{A}^P) = \pi(\mathcal{A}^{SP})$.

Appendix B: Extended Discussion

Discussion on Auction Reliability

Auction reliability, defined as the probability that the winning bidder fulfills their payment obligations, is a critical metric in economic terms as it directly impacts the seller's expected revenue and allocative efficiency. High reliability ensures that the auction outcome reflects the true valuation of goods, minimizing dead-weight losses [Ferrell, 2014] and enhancing economic efficiency. From the seller's perspective, reliability reduces transaction delays and re-auctioning costs, thus maximizing net revenues. Furthermore, it bolsters market trust, as credible auctions attract serious bidders, foster competition, and sustain long-term participation. Low reliability, by contrast, risks adverse selection, where less credible bidders dominate, leading to inefficiencies and reduced revenues. Such risks often necessitate enforcement mechanisms like escrow accounts or performance bonds to align incentives and safeguard transactions, as discussed in [Klemperer, 2004]. These measures, while potentially increasing upfront costs, are critical for ensuring the auction's credibility and success in high-stakes markets.

Discussion on Seller's Opportunity Cost

The opportunity cost loss for the seller when the winning bidder defaults in an auction represents a significant economic inefficiency, impacting the seller's expected utility and market dynamics. In economic terms, opportunity cost is the value of the next best alternative foregone. In the context of auctions, this cost is not merely the loss of the winning bid but also includes the potential revenue from the second-highest bid, which may not materialize under default conditions. For instance, in *real estate auctions*, when the winning bidder defaults, the seller must often relist the property, incurring holding costs such as taxes, maintenance, and depreciation [Vojtech et al., 2016]. Additionally, market conditions may deteriorate, forcing the seller to accept a lower price in subsequent auctions. Similarly, in *public procurement auctions* [Birulin, 2020], the default of a contractor selected to build infrastructure like roads or bridges can cause project delays, leading to increased material costs, labor expenses, and public discontent due to the delayed utility of the project. In *commodity auctions* [Bruce et al., 2004], such as for agricultural products, a defaulting buyer may result in perishable goods being wasted or sold at distress prices, further amplifying opportunity costs. For sellers of unique items, like art or rare collectibles, a default can result in the asset being relisted in a less competitive market or during an off-peak period, significantly reducing its expected sale price. This corresponds to the findings of [Ucbasaran et al., 2013], which argue that the consequences of a firm's initiative failure extend well beyond the immediate loss of revenue, often encompassing long-term psychological, social, and financial challenges. Therefore, these examples highlight that opportunity costs are not confined to the immediate loss of revenue but extend to the broader economic implications of delays, reputational damage, and market inefficiencies. This underscores the importance of auction mechanisms designed to mitigate default risks, incorporate opportunity costs into their structure, and prioritize bidder reliability to protect seller interests and ensure robust auction outcomes.

Discussion on Remark 1

In relatively stable markets, the default probability p is typically low, whereas under highly unstable conditions p may approach 1. The extant literature (e.g., [Lee and Li, 2019]) often addresses bidder default by imposing penalties upon default. However, when the default probability is driven by exogenous factors beyond the bidder's control, such penalties are a poor deterrent: they do not materially change p. Accordingly, implementing a penalty clause for defaults in such situations is unlikely to be effective in reducing the default probability p. Consider a private value sealed-bid auction with n risk-neutral bidders, each having a default probability p, which is exogenous and beyond their control. Therefore, if the seller introduces a penalty η for default, the payoff of a bidder (π_b) with private value v and bid b is given by:

$$\pi_b = (1 - p)F^{n-1}(b)(v - b) - pF^{n-1}(b)\eta$$

For a bidder to participate in the auction, the following condition must hold:

$$\pi_b \geq 0 \implies \eta \leq \frac{1-p}{p}$$

When the default probability p approaches 1, i.e., $p \to 1^-$, the penalty η must be less than 0^+ to ensure bidder participation. This indicates that introducing a penalty clause has minimal effect in scenarios where the default probability is predominantly high and outside the bidders' control. Therefore, in rest of the paper we are not going to consider penalty for the bidders for default. Our objective is to focus on designing and analyzing auction mechanisms that remain effective even when bidders face significantly high default probabilities.

Discussion on Remark 2

This setting rests on the observation made in [Sharma et al., 2014] that most of the firms are subject to market risk and uncertainties because they operate in dynamic economic environments where external factors influence their outcomes. Demand-side risks arise from fluctuations in consumer preferences, income levels, and macroeconomic conditions, impacting sales and revenues. Supply-side risks stem from volatility in input prices, labor costs, and technology changes, affecting production efficiency and costs. Additionally, firms face systematic risks like interest rate changes, inflation, or exchange rate volatility, which affect the broader economy. Such risks, being exogenous, cannot be fully diversified away and impact profitability and the valuation of the firms. Conversely, the firms that are somewhat insulated from market uncertainties are rare because most operate in interconnected economic systems where exogenous factors like demand shifts, supply chain dynamics, and macroeconomic variables affect outcomes. Complete insulation requires unique conditions, such as monopolistic power, highly inelastic demand for products (e.g., essential goods or utilities), or strong government protections like subsidies or tariffs. Even these firms face risks from broader systemic shocks, regulatory changes, or technological disruptions, making total insulation from market uncertainties very rare in competitive economies [Moyer and Chatfield, 1983]. Moreover, the assumption that $n-1$ bidders may default with probability p is consistent with the assumption in [Sebastian Perez-Salazar, 2024], which examines a sequential decision-making problem. We assume that the default probability of a bidder is a common knowledge across bidders and the seller.