Evaluation of Solving Methods for the Fundamental Matrix Computation

Katherine Arnold — Vision and Image Processing Lab, University of Waterloo, Waterloo, ON, Canada
Mohamed A. Naiel — Vision and Image Processing Lab, University of Waterloo, Waterloo, ON, Canada
Mark Lamm — Christie Digital Systems Canada Inc., Kitchener, ON, Canada *
Paul Fieguth — Vision and Image Processing Lab, University of Waterloo, Waterloo, ON, Canada
Email:{k2arnold, mohamed.naiel, pfieguth}@uwaterloo.ca, mark.lamm@christiedigital.com

Abstract

Solving the fundamental matrix is a key step in many image calibration and 3D reconstruction systems. The goal of this paper is to study the performance of non-linear solvers for estimating the fundamental matrix in projector-camera calibration. To prevent measurements errors from distorting our understanding, synthetic data are created from ground-truth camera and projector parameters and then used for the assessment of four nonlinear solving strategies.

1 Introduction

Projector-Camera systems are now widely used in many applications, including 3D reconstruction and projection mapping [1, 2]. The goal of these systems is to estimate distortion parameters via the so-called fundamental matrix, which includes the intrinsic and extrinsic parameters of the projector(s) and camera(s) for correct geometric projection to/from a given surface. The fundamental matrix F relates point correspondences in two different image views of the same 3D scene. Therefore, estimating the fundamental matrix through point correspondences allows for the inference of the needed intrinsic and extrinsic parameters embedded/implied in F. F must satisfy

$$x_c^\top F x_p = 0, \tag{1}$$

where x_c, x_p are corresponding points in the camera and projector planes, respectively. In [1], Li *et al.* have utilized a greedy minimization algorithm with the following objective function in order to estimate the fundamental matrix:

$$\Psi = C_G(F) + \lambda_p C_p(x_c', x_p') + \lambda_f C_f(f_c, f_p). \tag{2}$$

Their proposed objective function explicitly asserts prior models for the principal points (x_c', x_p') and focal lengths (f_c, f_p), and then a standard normalized 8-point algorithm with RANSAC to propose an estimate of the fundamental matrix based on C_G, the Gold-Standard reprojection error.

This paper aims to investigate the solving strategies that produce the least residual error in estimating the fundamental matrix, where the performance of the solving methods are assessed using the established objective function in (2). The error inherent in the computation method is evaluated by employing the use of exact synthetic data with known ground truth.

2 Experimental Evaluation

Evaluation Methodology: Four non-linear solving methods are selected to compare the performance of the system: (a) Levenberg–Marquardt , which was originally used in the baseline algorithm of [1, 3], (b) Quasi-Newton[1] [3], (c) Trust Region [3], and (d) Trust Region Reflective [3]. These methods are chosen for their computational efficiency, ability to handle unconstrained minimization, and robustness to initial estimate variations. Each method is tested with the same priors and cost function of (2).

New synthetic data is generated using a 3D predefined model, for which Figure 1 illustrates an example of 3D shape and poses for the projector and camera. The intrinsic parameters of a projector-camera setup are selected and used to compute the ground-truth matrices. For convenience, the projector centre is set to be the origin, so the camera extrinsic parameters and baseline are measured with respect to the projector. The 3D CAD model of the scene is converted into a 3D point cloud from which points are sampled and then reprojected into the image planes of the camera and projector:

$$x_i^\top = P_i^\top X_{\text{world}} \tag{3}$$

*This research would not be possible without the support of the Natural Sciences and Engineering Research Council of Canada (NSERC-CRD) and Christie Digital Systems Inc.

[1] Quasi-Newton with Broyden–Fletcher–Goldfarb–Shanno algorithm was used.

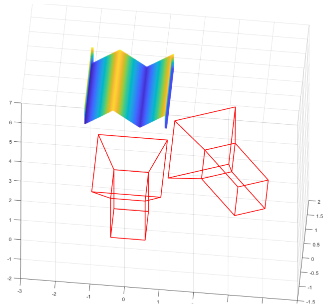

Fig. 1: The proposed Projector-Camera scene for synthetic data capture.

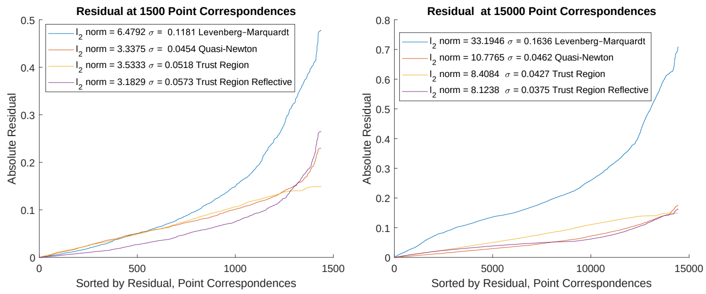

Fig. 2: One case comparing the residual magnitude values for the different solvers at 1500 (left) and 15000 (right) point correspondences.

P is the projection matrix and index i indicates either a camera or projector. The generated point correspondences are used as the input to the objective function (2), from which each of the solvers is used to estimate the fundamental matrix. To assess the performance of the solvers, the residual error of the epipolar geometry (1) is used, where it can be computed using the Kronecker product, $\otimes$, and the vectorized version of the fundamental matrix as [4]:

$$\text{Residual} = (x_c \otimes x_p)^\top \text{vec}(F) \tag{4}$$

Results: The four selected solvers are compared in terms of the Residual reprojection error (4) using sets of 1500 and 15000 synthetic corresponding points. As shown in Figure 2, we observe the Levenberg-Marquardt method producing the highest l_2-norm and standard deviation of the absolute residual error over the data sets, with the Trust Region Reflective method producing the lowest, which suggests there is still significant room for improving the performance the baseline 3D reconstruction. Consistent conclusions are reached when the same experiment is repeated with different point-correspondences and initial estimate of F. Future work includes exploring the sensitivity to noise and assessing performance dependency on the accuracy of the initial (prior) estimates.

References

[1] F. Li, H. Sekkati, J. Deglint, C. Scharfenberger, M. Lamm, D. Clausi, J. Zelek, and A. Wong, “Simultaneous projector-camera self-calibration for three-dimensional reconstruction and projection mapping,” *IEEE Transactions on Computational Imaging*, vol. 3, no. 1, pp. 74–83, 2017.

[2] B. Huang, Y. Tang, S. Ozdemir, and H. Ling, “A fast and flexible projector-camera calibration system,” *IEEE TASE*, 2020.

[3] W. Sun and Y.-X. Yuan, *Optimization theory and methods: nonlinear programming*. Springer Science & Business Media, 2006, vol. 1.

[4] W. Förstner and B. P. Wrobel, *Photogrammetric computer vision*. Springer, 2016.