Evaluating the Impact of Stereo Overlap on Calibration parameters in Camera and Projector Systems

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Abstract

Stereo overlap plays a key role in projection mapping, influencing calibration accuracy and system cost. Understanding the relationship between stereo overlap and calibration error allows for reducing overlap while maintaining acceptable accuracy, thus reducing the number of cameras needed and cutting costs. This paper investigates the impact of stereo overlap on camera and projector calibration using synthetic data. The first study examines camera calibration accuracy as a function of stereo overlap between cameras, while the second extends the analysis to projectors. The findings contribute to insight in reducing costs while maintaining system accuracy.

1 Introduction

Display projectors are integral to a wide range of applications, from theaters to sports arenas and other public venues [1]. In these setups, multiple (possibly many) projectors are used to divide and project content across large surfaces. Ensuring smooth transitions and precise alignment between projections is critical for delivering a cohesive and visually meaningful display. The proper calibration of projectors is essential, as it directly affects the quality of the final 3D projection [2]. Calibration entails determining the intrinsic and extrinsic parameters of cameras and projectors to ensure accurate alignment of projections and images in three-dimensional space.

Traditionally, achieving high calibration accuracy required complex setups, particularly for large-scale projection systems. Researchers have extensively studied the impact of noise on calibration parameters [3], however there are factors present other than noise, and the influence the degree of stereo overlap between cameras, as well as the overlap between projectors and cameras, remains inadequately explored in terms of its impact on the resulting calibration accuracy.

Understanding the role of stereo overlap is important for balancing calibration accuracy with cost-effectiveness. If it is realistically possible to significantly reduce the stereo overlap, then setup costs can be minimized, but clearly too little an overlap may lead to inferior calibration. Thus, analyzing calibration error in relation to stereo overlap allows us to determine an optimal overlap level, which is the subject of this paper.

The rest of the paper is organized as follows. Section 2 provides the background, followed by Section 3, which describes the data used. Section 4 explains the methodology of the analysis. Section 5 presents the results, and Section 6 concludes the paper.

2 Background

Calibration involves determining both the intrinsic parameters (K) and extrinsic parameters (A) of each device (camera or projector). Intrinsic parameters describe the internal characteristics of the camera (K_c) or projector (K_p), such as focal length, principal point, and skew [4, 5]. The intrinsic matrix of the pinhole camera model is defined as

$$K = \begin{bmatrix} f_x & s & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

where f_x and f_y are the focal lengths in pixel units, p_x and p_y represents the coordinates of the principal point, and s is the skew coefficient.

Extrinsic parameters describe the position and orientation of the camera (A_c) or projector (A_p) with respect to the reference frame (typically chosen, arbitrarily, as one of the cameras). The extrinsic matrix can be expressed as

$$A_c = \begin{bmatrix} R_c & | & \underline{t}_c \end{bmatrix} \quad A_p = \begin{bmatrix} R_p & | & \underline{t}_p \end{bmatrix} \qquad (2)$$

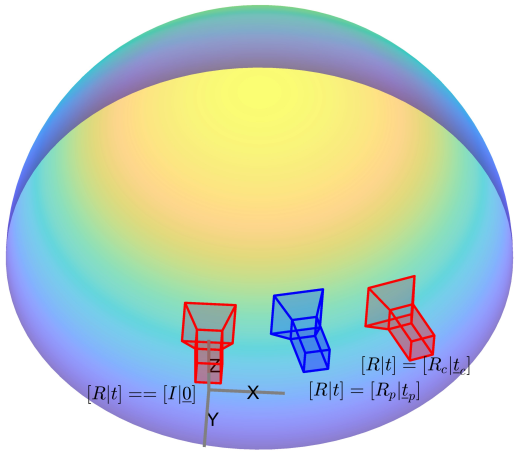

Figure 1: Simulation setup with red representing cameras and blue representing projector, all projecting onto the hemispherical surface.

3 Data

Simulation data is generated for this analysis. A hemispherical dome is used as the (unknown) 3D surface observed by the cameras and displayed onto by the projectors. Initially, the correspondences between the camera image points and the world points are identified by multiplying the projection matrix with the world points. This transformation is represented by

$$\underline{u}_c = P_c \cdot \underline{u}_w \quad P_c = K_c \cdot A_c \tag{3}$$

where $\underline{u}_c$ represents the image points in the camera plane, $\underline{u}_w$ represents the world points, and P_c is the projection matrix of the camera. Subsequently, correspondences between Camera 1 and Camera 2 are identified by matching the world points projected onto the respective image planes, and designated as $(\underline{u}_{c1}, \underline{u}_{c2})$.

Additionally, the transformation described by Eqn. 4 is applied to the projector to establish correspondences between the world points and the image points on the projector's image plane. Finally, the correspondences between the projector and the cameras are established by matching the world points in the projector and camera image planes, and designated as $(\underline{u}_{c1}, \underline{u}_p)$ and $(\underline{u}_{c2}, \underline{u}_p)$.

$$\underline{u}_p = P_p \cdot \underline{u}_w \quad P_p = K_p \cdot A_p \tag{4}$$

where $\underline{u}_p$ represents the image points in the projector plane, and P_p is the projection matrix of the projector. To enhance the realism of the synthetic data, noise (ϵ) is added to the correspondences. ϵ is a random variable which follows a normal distribution with a mean of zero and a standard deviation of σ and is expressed as:

$$\epsilon \simeq \mathcal{N}(0, \sigma^2) \tag{5}$$

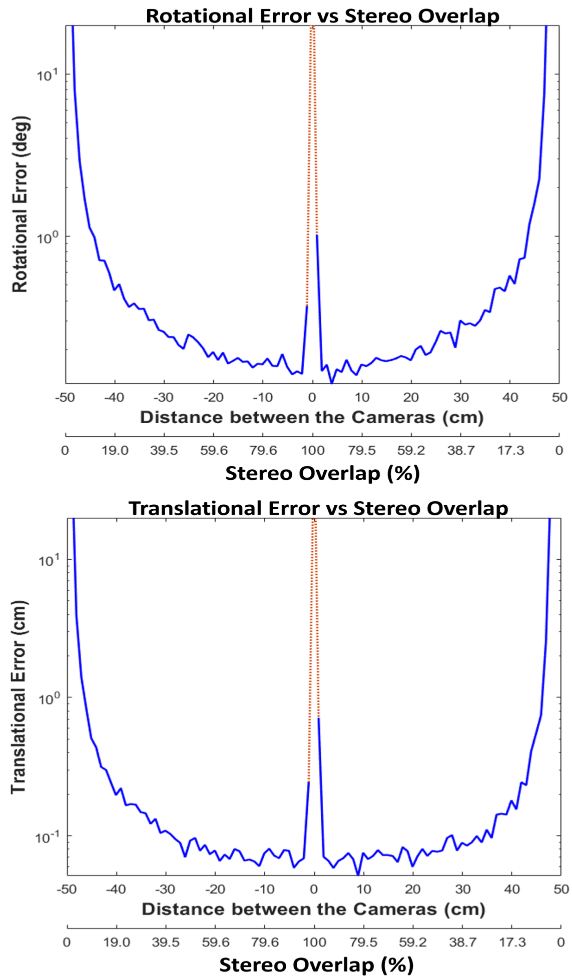

Figure 2: Error plots for Study 1. Errors generally increase as stereo overlap decreases. At 100% overlap, close camera placement causes numerical instability, indicated by red dotted lines.

where the value of σ is considered to be 1 in this study.

For both studies, K_c and K_p are assigned standard values at the outset. Additionally, K_{c1} and K_{c2} are defined to be identical.

$$K_c = K_{c1} = K_{c2} \tag{6}$$

4 Methodology

4.1 Study 1: Camera Calibration Accuracy Based on Stereo Overlap

To analyze the accuracy of camera calibration (A_{c2}) in relation to stereo overlap (α) between the cameras, t_{c2} is incrementally moved away from the camera in both the negative and positive directions, thereby varying the stereo overlap from 100 to 0. The value of α is calculated as the total area in the camera plane that is viewed by both cameras divided by the combined image plane

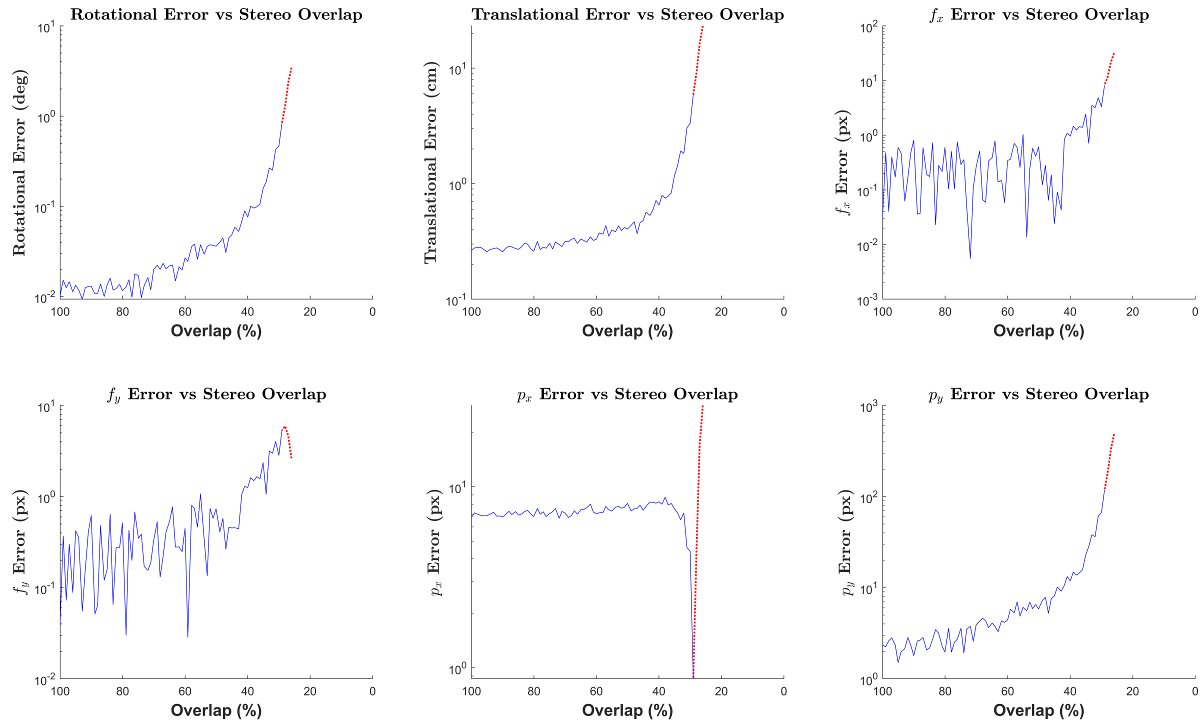

Figure 3: Error plots for Study 2. Errors increase as stereo overlap decreases, with red dotted lines marking reprojection errors exceeding 3 pixels, where calibration accuracy becomes unacceptable. Calibration is not computed below 20% overlap due to insufficient correspondences.

area of both cameras. With the variation of α, values of R_{c2}, and t_{c2} are determined and compared against ground truth values.

For this analysis, K_c, A_{c1} and ($\underline{u}_{c1}$, $\underline{u}_{c2}$) are known, while A_{c2} is estimated. To identify A_{c2}, the essential matrix (E) [5] is first computed using the normalized point correspondences [5] ($\hat{\underline{u}}_{c1}$, $\hat{\underline{u}}_{c2}$), as shown in Eq. 7. This matrix is then decomposed using Singular Value Decomposition (SVD) [6], allowing for the extraction of A_{c2} and t_{c2}.

$$\hat{\underline{u}}_{c2}^T \cdot E \cdot \hat{\underline{u}}_{c1} = 0 \tag{7}$$

For each value of stereo overlap, the output values are computed 40 times, and the resulting errors are averaged to reduce fluctuations.

4.2 Study 2: Projector Calibration Accuracy Based on Stereo Overlap

To evaluate the precision of projector calibration (K_p and A_p) as a function of stereo overlap (β) between the cameras and projectors, t_{c2} is incrementally moved away from the camera in both the negative and positive directions, thereby varying the β from 0 to 100. The value of β is calculated as the area of the projector region visible to both cameras, divided by the total area of the projector in the projector image plane. As β is varied, the values of K_p and A_p are determined and compared with ground truth values.

For this analysis, K_c, A_c and ($\underline{u}_c$, $\underline{u}_p$) are known, while K_p and A_p are estimated. Initially, triangulation [7, 5] is performed with cameras to identify the world points ($\underline{u}_w$) in the stereo region of the cameras and is seen by the projector.

Next, using the identified ($\underline{u}_w$) and the known $\underline{u}_p$, the projector's projection matrix P_p is determined. Finally, P_p is decomposed using RQ decomposition [8] to obtain the K_p and A_p. The simulation setup for Study 1 and 2 is shown in Fig. 1.

5 Results

To evaluate calibration results, specific error metrics were applied to each parameter. For translational errors, Euclidean distance was calculated between the estimated and ground truth positions, while absolute difference was used to compute the error value for other parameters across both studies.

The results reveal that errors increase as stereo overlap decreases. In the calibration of cameras (see Figure 2), errors rise sharply when the stereo overlap falls below 15%. Additionally, errors also increase near 100% overlap, where the cameras are placed close together, leading to numerical instability. In the calibration of projectors (see Figure 3), a similar pattern is observed, with errors increasing as the stereo overlap decreases. Notably, after 20% overlap, projector calibration becomes impossible due to insufficient correspondences. In study 2, the dotted lines in the plots represent instances where the reprojection error exceeds 3px, a level that is unacceptable for realistic projections.

6 Conclusion and Future Work

This paper highlights the influence of stereo overlap on the accuracy of camera and projector calibration, revealing that calibration errors significantly increase as overlap diminishes, particularly below 15% for cameras and 20% for projectors. Moreover, even with 100% overlap, calibration errors remain significantly high when the distance between devices approaches zero due to numerical instability. Future work will focus on testing these methods in real-world scenarios.

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