A Discretize-then-Optimize Approach to Super-Resolution Reconstruction and Motion Estimation

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1 Abstract

The process of recovering a high-resolution (HR) image from a set of distorted (i.e. deformed, blurry, noisy, etc.) low-resolution (LR) images is known as *super-resolution*. *Super-resolution* problem will require the reconstruction of the HR image and estimations of motion between LR images. In this study, image reconstruction and motion estimation will be treated as a coupled problem. The proposed algorithm uses an inverse model followed by a discretize-then-optimize approach. Preliminary experiments on test data will be presented.

2 Mathematical Formulation

2.1 Problem Statement

Assume that all images are represented in the continuous domain. Given a set of m LR images, $y = \{y_1, \dots, y_m\}$, y_i: $\mathbb{R}^2 \to \mathbb{R}$, we wish to find a HR image f: $\Omega \subset \mathbb{R}^2 \to \mathbb{R}$, where Ω is the region of interest, and a set of transformations $u = \{u_1, \dots, u_m\}$, u_i: $\mathbb{R}^2 \to \mathbb{R}^2$, that minimizes the objective functional

$$\mathcal{J}[u,f] = \sum_{i=1}^{m} \mathcal{D}\big[y_i, \mathcal{H}f[u_i]\big] + \sum_{i=1}^{m} \alpha_i \mathcal{S}[u_i - u^{ref}] + \beta\,\mathcal{Q}[f], \tag{1}$$

where $\mathcal{D}\big[y_i,\mathcal{H}f[u_i]\big]$ is a distance measure between y_i and $\mathcal{H}f[u_i]$ defined as sum of squared differences, $\mathcal{S}[u_i\text{-}u^{ref}]$ is the elastic regularizer, and $\mathcal{Q}[f]$ is the total variation penalty on the computed image f. $\alpha = \{\alpha_i,\dots,\alpha_m\} \in \mathbb{R}^m$ and $\beta \in \mathbb{R}$ are regularization parameters. It is assumed that $u^{ref}(x)$=x where x is the identity transformation. $\mathcal{H}$ is the composition of blur and downsampling operators.

2.2 Discretization

Let D, S and Q represent the discretized counterparts of the distance measure ($\mathcal{D}$), elastic ($\mathcal{S}$) and total variation ($\mathcal{Q}$) regularizers respectively. H is the discretized degradation operator, and corresponding images and grids will now be represented by discrete vectors y_i, f, and u_i. The discretization of the model can be obtained by a slight modification to the objective functional

$$J[\mathrm{u},\mathrm{f}] = \sum_{i=1}^{m} D\big[\mathrm{y}_i, \mathrm{Hf}[\mathrm{Pu}_i]\big] + \sum_{i=1}^{m} \alpha_i S[\mathrm{u}_i - \mathrm{u}^{\mathrm{ref}}] + \beta Q[\mathrm{f}]. \tag{2}$$

Note that the linear operator P was introduced to preserve grid consistency, as elastic regularization is defined on staggered grid while pixel intensity values are recorded at cell-centers.

2.3 Optimization

The objective functional will be optimized using ℓ-BFGS. Partial derivatives of the objective functional are computed as ℓ-BFGS requires the Jacobian to be explicitly defined. Let r_i = y_i - $\mathrm{Hf}[\mathrm{Pu}_i]$, then

$$\begin{aligned}\frac{\partial J}{\partial \mathrm{u}_i} &= \frac{\partial D}{\partial r_i}\frac{\partial r_i}{\partial \mathrm{f}}\frac{\partial \mathrm{f}}{\partial(\mathrm{Pu}_i)}\frac{\partial(\mathrm{Pu}_i)}{\partial \mathrm{u}_i} + \alpha_i\frac{\partial S}{\partial \mathrm{u}_i} \\ &= (r_i^T)(-\mathrm{H})\left(\frac{\partial \mathrm{f}}{\partial(\mathrm{Pu}_i)}\right)(\mathrm{P}) + \alpha_i\frac{\partial S}{\partial \mathrm{u}_i}\end{aligned} \tag{3}$$

for all i = 1, $\dots$, m, and

$$\begin{aligned}\frac{\partial J}{\partial \mathrm{f}} &= \sum_{i=1}^{m}\frac{\partial D}{\partial r_i}\frac{\partial r_i}{\partial \mathrm{f}} + \beta\frac{\partial Q[\mathrm{f}]}{\partial \mathrm{f}} \\ &= \sum_{i=1}^{m}(r_i^T)(-\mathrm{H}) + \beta dQ[\mathrm{f}].\end{aligned} \tag{4}$$

3 Experimental Results

The experiment uses a 60-frame sequence of resolution 38x34. Fig. 1 displays a comparison between noiseless and noisy frames using the parameters α = 300, β = 75 with 100 iterations. Noisy frames are generated from the same image sequence by applying a zero mean additive white Gaussian noise of σ = 1.

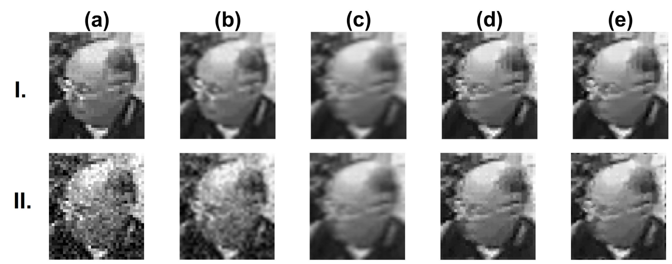

Fig. 1: Experimental results using 60 frames of noiseless (Row I) and noisy (Row II) images with a zooming factor of 2. (a) y_{26} (Source frame #26) (b) y_{26} 2x resolution (bilinear interpolation) (c) Average of y_i (d) Computed image f (e) f[Pu_{26}]

4 Conclusion and Future Work

Preliminary experiments show that results produced using noisy inputs are comparable to the results produced by its noiseless counterparts. Computed results tend to be highly dependent on the initial condition and values/ratio of α and β. Techniques such as multi-level image registration may assist in determining more suitable initial conditions, while image reconstruction and motion detection may be decoupled to reduce the difficulty in finding optimal values of these parameters. Multi-modal super-resolution may also be explored as the proposed algorithm provides a flexible framework that allows simple implementation of different regularization schemes.

Acknowledgments

This research was supported in part by NSERC through an NSERC USRA for Eric Ng and an NSERC Discovery Grant for Mehran Ebrahimi.

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